EE2030Linear Algebra

Homework#5

reference solution

1.

The first determinant is 0. The second is $1 - 2t^2 + t^4 = (1 - t^2)^2$

2.

Here $A = LU$ with $\det(L) = 1$ and $\det(U) = -6$ = product of pivots, so also $\det(A) = -6$. $\det(U^{-1}L^{-1}) = -\frac{1}{6} = 1/\det(A)$ and $\det(U^{-1}L^{-1}A)$ is $\det I = 1$.

3.

(a) *True*: $\det(AB) = \det(A)\det(B) = 0$ (b) *False*: A row exchange gives $-\det =$ product of pivots. (c) *False*: $A = 2I$ and $B = I$ have $A - B = I$ but the determinants have $2^n - 1 \neq 1$ (d) *True*: $\det(AB) = \det(A)\det(B) = \det(BA)$.

4.

(a) $C_1 = 0,\ C_2 = -1,\ C_3 = 0,\ C_4 = 1$ (b) $C_n = -C_{n-2}$ by cofactors of row 1 then cofactors of column 1. Therefore $C_{10} = -C_8 = C_6 = -C_4 = C_2 = -1$.

5.

$G_2 = -1$, $G_3 = 2$, $G_4 = -3$, and $G_n = (-1)^{n-1}(n-1)$. One way to reach that G_n is to multiply the n eigenvalues $-1, -1, \ldots, -1, n-1$ of the matrix.

6.

$\det P = -1$ because the cofactor of $P_{14} = 1$ in row one has sign $(-1)^{1+4}$. The big formula for $\det P$ has only one term $(1 \cdot 1 \cdot 1 \cdot 1)$ with minus sign because three exchanges take $4, 1, 2, 3$ into $1, 2, 3, 4$; $\det(P^2) = (\det P)(\det P) = +1$ so

$$\det \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} = \det \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ is } \textit{not right}.$$

7.

$$C = \begin{bmatrix} 6 & -3 & 0 \\ 3 & 1 & -1 \\ -6 & 2 & 1 \end{bmatrix} \text{ and } AC^{\mathrm{T}} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

This is $(\det A)I$ and $\det A = 3$.
The $1, 3$ cofactor of A is 0.
Then $C_{31} = 4$ or 100: no change.

8.

$\begin{vmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{2} & \mathbf{3} \end{vmatrix} = 4 = \begin{vmatrix} \mathbf{2} & \mathbf{2} \\ \mathbf{1} & \mathbf{3} \end{vmatrix}$ because the transpose has the same determinant.

9.

The box has height 4 and volume $= \det \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 4 \end{bmatrix} = 4$. $\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k}$ and $(\boldsymbol{k} \cdot \boldsymbol{w}) = 4$.