EE2030Linear Algebra

homework#4

Reference solution

1.

The vector = $c(5,6,5)^T$

2.

If $\boldsymbol{S}$ is the subspace of $\mathbf{R}^3$ containing only zero vector, then $\boldsymbol{S}^\perp$ is all of $\mathbf{R}^3$.
If $\boldsymbol{S}$ is spanned by (1,1,1), then $\boldsymbol{S}^\perp$ is the plane spanned by (1,-1,0) and (1,0,-1) or any subspace with its basis is orthogonal to the (1,1,1).
If $\boldsymbol{S}$ is spanned by (1,1,1) and (1,1,-1), then $\boldsymbol{S}^\perp$ is the line spanned by (1,-1,0) or any subspace with its basis is orthogonal to the (1,1,1) and (1,1,-1).

3.

(a) 兩平面必有交線，不可能是 orthogonal spaces

(b) (0,0,1,1,1)和(2,-2,3,4,-4)內積不等於 0

(c) 反例:由(1,0)和(1,1)所構成知兩直線，meet only in the zero vector but not orthogonal.

4.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, P = \text{square matrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \boldsymbol{p} = P \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \end{bmatrix}.$$

5.

$(I-P)^2=I^2-2P+P^2=I-P$,I-P projects onto the $N(A^T)$

6.

$$\frac{1}{36}\begin{bmatrix}p_1^T\\p_2^T\\p_3^T\end{bmatrix}\begin{bmatrix}p_1 p_2 p_3\end{bmatrix}=\frac{1}{36}\begin{bmatrix}p_1^Tp_1 & p_1^Tp_2 & p_1^Tp_3\\p_2^Tp_1 & p_2^Tp_2 & p_2^Tp_3\\p_3^Tp_1 & p_3^Tp_2 & p_3^Tp_3\end{bmatrix}$$

$P_{1,2,3}$ is column vector

7.

(a)算出 line:b=1+4t，帶入檢查

(b)略

8.

(a) $\boldsymbol{a} = (1,\ldots,1)$ has $\boldsymbol{a}^{\mathrm{T}}\boldsymbol{a} = m$, $\boldsymbol{a}^{\mathrm{T}}\boldsymbol{b} = b_1 + \cdots + b_m$. Therefore $\widehat{x} = \boldsymbol{a}^{\mathrm{T}}\boldsymbol{b}/m$ is the **mean** of the b's (their average value)

(b) $\boldsymbol{e} = \boldsymbol{b} - \widehat{x}\boldsymbol{a}$ and $\|\boldsymbol{e}\|^2 = (b_1 - \text{ mean })^2 + \cdots + (b_m - \text{ mean })^2 =$ **variance** (denoted by $\boldsymbol{\sigma^2}$).

(c) $\boldsymbol{p} = (3,3,3)$ and $\boldsymbol{e} = (-2,-1,3)$ $\boldsymbol{p}^{\mathrm{T}}\boldsymbol{e} = 0$. Projection matrix $P = \frac{1}{3}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$.

9.

The least squares equation is $\begin{bmatrix}5 & \mathbf{0}\\ \mathbf{0} & 10\end{bmatrix}\begin{bmatrix}C\\D\end{bmatrix} = \begin{bmatrix}5\\-10\end{bmatrix}$. Solution: $C = 1$, $D = -1$.
The best line is $b = 1 - t$. Symmetric t's $\Rightarrow$ diagonal $A^{\mathrm{T}}A \Rightarrow$ easy solution.

10.

$\boldsymbol{A} = \boldsymbol{a} = (1,-1,0,0)$; $\boldsymbol{B} = \boldsymbol{b} - \boldsymbol{p} = (\frac{1}{2},\frac{1}{2},-1,0)$; $\boldsymbol{C} = \boldsymbol{c} - \boldsymbol{p}_A - \boldsymbol{p}_B = (\frac{1}{3},\frac{1}{3},\frac{1}{3},-1)$.
Notice the pattern in those orthogonal $\boldsymbol{A},\boldsymbol{B},\boldsymbol{C}$. In $\mathbf{R}^5$, $\boldsymbol{D}$ would be $(\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4},-1)$.
Gram-Schmidt would go on to normalize $\boldsymbol{q}_1 = \boldsymbol{A}/||\boldsymbol{A}||$, $\boldsymbol{q}_2 = \boldsymbol{B}/||\boldsymbol{B}||$, $\boldsymbol{q}_3 = \boldsymbol{C}/||\boldsymbol{C}||$.

11.

(a) (-1,1,0,0),(-1,0,1,0),(1,0,0,1)

(b) (1,1,1,-1)

(c) b_1=(0.5,0.5,0.5,1.5),b_2=(0.5,0.5,0.5,-0.5)

12.

(a) $c = \frac{1}{2}$ normalizes all the orthogonal columns to have unit length (b) The projection $(\boldsymbol{a}^{\mathrm{T}}\boldsymbol{b}/\boldsymbol{a}^{\mathrm{T}}\boldsymbol{a})\boldsymbol{a}$ of $\boldsymbol{b} = (1,1,1,1)$ onto the first column is $\boldsymbol{p}_1 = \frac{1}{2}(-1,1,1,1)$. (Check $\boldsymbol{e} = \mathbf{0}$.) To project onto the plane, add $\boldsymbol{p}_2 = \frac{1}{2}(1,-1,1,1)$ to get $(0,0,1,1)$.