EE2030Linear Algebra

Homework#3

Due: 03/29/2023 10:10(Wed)

1. (a)Solvable if $b_2 = 2b_1$ and $3b_1 - 3b_3 + b_4 = 0$. Then $x = \begin{bmatrix} 5b_1 - 2b_3 \\ b_3 - 2b_1 \end{bmatrix} = x_p$

   (b) Solvable if $b_2 = 2b_1$ and $3b_1 - 3b_3 + b_4 = 0$. $x = \begin{bmatrix} 5b_1 - 2b_3 \\ b_3 - 2b_1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$.

2. Rank = 2; rank = 3 unless q = 2(then rank = 2). Transpose has the same rank!

3. $$\begin{bmatrix} 1 & 0 & 2 & 3 & \mathbf{2} \\ 1 & 3 & 2 & 0 & \mathbf{5} \\ 2 & 0 & 4 & 9 & \mathbf{10} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 3 & \mathbf{2} \\ 0 & 3 & 0 & -3 & \mathbf{3} \\ 0 & 0 & 0 & 3 & \mathbf{6} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 2 & 0 & \mathbf{-4} \\ 0 & 1 & 0 & 0 & \mathbf{3} \\ 0 & 0 & 0 & 1 & \mathbf{2} \end{bmatrix}; \begin{bmatrix} -4 \\ 3 \\ 0 \\ 2 \end{bmatrix}; x_n = x_3 \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

4. Columns 1 and 2 are bases for the (**different**) column spaces of A and U; rows 1 and 2 are bases for the (**equal**) row spaces of A and U; (1, 1, 1) is a basis for the (**equal**) nullspaces.

5. A has rank 2 if $c = 0$ and $d = 2$; $B = \begin{bmatrix} c & d \\ d & c \end{bmatrix}$ has rank 2 except when $c = d$ or $c = -d$.

6. The dimension of **S** spanned by all rearrangements of **x** is
   (a) zero, when $\mathbf{x} = \mathbf{0}$
   (b) one, when $\mathbf{x} = (1, 1, 1, 1)$
   (c) three, consider a non-zero vector (x_1, x_2, x_3, x_4) such that $x_1+x_2+x_3+x_4 = 0$, i.e., its dot product with (1, 1, 1, 1) is 0.
   (d) four when the **x**'s are not equal and don't add to zero. **No x gives** dim **S** = 2.

7. Row space basis can be the nonzero rows of U: (1, 2, 3, 4), (0, 1, 2, 3), (0, 0, 1, 2); nullspace basis (0, 1, 2, 1) as for U; column space basis (1, 0, 0), (0, 1, 0), (0, 0, 1) (happen to have $\mathbf{C}(A) = \mathbf{C}(U) = \mathrm{R}^3$); left nullspace has empty basis.

8. row(3) - 2row(2) + row(1) = $\vec{0}$ so the vectors c(1, -2, 1) are in the left nullspace. The same vectors happen to be in the nullspace (an accident for this matrix).

9. (a) $AX = 0$, if each column of X is a multiple of (1, 1, 1); dim(nullspace) = 3.
   (b) If $AX = B$ then all columns of B add to zero; dimension of the B's = 6.
   (c) $3 + 6 = \dim(M^{3\times 3}) = 9$ entries in a 3 by 3 matrix