EE2030Linear Algebra

homework#2

Reference solution

1.

The only subspaces are (a) the plane with b1 = b2 (d) the linear combinations of v and w (e) the plane with b1 + b2 + b3 = 0.

2.

The extra column $\boldsymbol{b}$ enlarges the column space unless $\boldsymbol{b}$ is *already* in the column space.

$$[A \quad \boldsymbol{b}] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(larger column space)
(no solution to $A\boldsymbol{x}=\boldsymbol{b}$)

$$[A \quad \boldsymbol{b}] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

($\boldsymbol{b}$ is in column space)
($A\boldsymbol{x}=\boldsymbol{b}$ has a solution)

3.

A $= \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ do not have $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ in C(A). A $= \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 3 & 6 & 0 \end{bmatrix}$ has C(A) = line in $\mathbf{R}^3$

4.

$x - 3y - z = 0$

$$\begin{bmatrix} 1 & -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = [0]$$

$x = 3y + z$ thus

$$x = \begin{bmatrix} 3y+z \\ y \\ z \end{bmatrix} = y \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

matrix A =[1 -3 -1]
x is the pivot variable and free variables are y z
special solutions:[3 1 0],[1 0 1]

5.

Fill in 12 then 4 then 1 to get the complete solution in $\mathbf{R}^3$ to $x - 3y - z = 12$:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \mathbf{12} \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} \mathbf{3} \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} \mathbf{1} \\ 0 \\ 1 \end{bmatrix} = x_{particular} + x_{nullspace}.$$

6.

Column 5 is sure to have no pivot since it is a combination of earlier columns. With 4 pivots in the other columns, the special solution is s = (1, 0, 1, 0, 1). The nullspace contains all multiples of this vector s (this nullspace is a line in R5
).

7.

This construction is impossible for 3 by 3 ! 2 pivot columns and 2 free variables.

8.

$$\mathrm{A} = \begin{bmatrix} 1 & 2 & 4 \\ 2 & \mathbf{4} & \mathbf{8} \\ 4 & \mathbf{8} & \mathbf{16} \end{bmatrix} \text{ and } \mathrm{B} = \begin{bmatrix} \mathbf{3} & 9 & -\frac{\mathbf{9}}{\mathbf{2}} \\ 1 & \mathbf{3} & -\frac{\mathbf{3}}{\mathbf{2}} \\ 2 & 6 & -3 \end{bmatrix} \text{ and } \mathrm{M} = \begin{bmatrix} a & b \\ c & \frac{\mathbf{bc}}{\mathbf{a}} \end{bmatrix}.$$

9.

$$A = \begin{bmatrix} 3 & 6 & 6 \\ 1 & 2 & 2 \\ 4 & 8 & 8 \end{bmatrix} = \boldsymbol{u}\boldsymbol{v}^T = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \end{bmatrix} \text{ and}$$

$$A = \begin{bmatrix} 2 & 2 & 6 & 4 \\ -1 & -1 & -3 & -2 \end{bmatrix} = \boldsymbol{u}\boldsymbol{v}^T = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 & 2 \end{bmatrix}$$

10.

$A = \begin{bmatrix} I & I \end{bmatrix}$ has $N = \begin{bmatrix} I \\ -I \end{bmatrix}$; $B = \begin{bmatrix} I & I \\ 0 & 0 \end{bmatrix}$ has the same N; $C = \begin{bmatrix} I & I & I \end{bmatrix}$ has $N = \begin{bmatrix} -I & -I \\ I & 0 \\ 0 & I \end{bmatrix}$.