EE205003 Linear Algebra, 2020 Fall Semester

Quiz # 12

DATE: Jan. 6th, 2021

1. (15%) Find an orthogonal basis for Span(S) where

$$S = \{(1, -2, 1, 0), (0, 1, 1, 0), (1, 0, 3, -1)\}$$

is a subset of $\mathbb{R}^4$.

2. (15%) Please find a least-squares solution of the following inconsistent linear system:

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & -2 & 1 \\ -2 & 2 & 3 \\ 4 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 3 \\ 0 \end{bmatrix}.$$

3. (15%) Do you expect these two systems to have the same least-squares solutions?

$$\begin{cases} 3x - 2y &= -5 \\ 2x + 3y &= 4 \\ x - 2y &= 3 \end{cases} \quad \text{and} \quad \begin{cases} 3x - 2y &= -5 \\ 2x + 3y &= 4 \\ 4x - 8y &= 12 \end{cases}$$

Explain why or why not.

4. (15%) Let $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ be an orthonormal set in an m-dimensional inner product space, where $m > k$. Explain why there is a vector $\mathbf{v}_{k+1}$ in the space such that $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k, \mathbf{v}_{k+1}\}$ is orthonormal.

5. (15%) Please show that any least-squares solution $\mathbf{x}$ of the real linear system $A\mathbf{x} = \mathbf{b}$ has the property $\mathbf{b}^T A\mathbf{x} \geq 0$.

6. An $n \times n$ complex matrix A is called nonnegative definite if $\mathbf{x}^H A\mathbf{x} \geq 0$ for all $\mathbf{x} \in \mathbb{C}^n$.

   (a) (5%) Please show that the diagonal elements a_{ii} of a nonnegative definite matrix A is nonnegative.

   (b) (5%) Any eigenvalue of a nonnegative definite matrix A is nonnegative.

   (c) (5%) Please show that if A is nonnegative definite, then $\langle A\mathbf{x}, \mathbf{x}\rangle = \langle \mathbf{x}, A\mathbf{x}\rangle$ for all $\mathbf{x} \in \mathbb{C}^n$, where $\langle \cdot, \cdot \rangle$ is the standard inner product for $\mathbb{C}^n$.

   (d) (5%) Please show that a nonnegative definite matrix A is Hermitian, i.e., $\langle A\mathbf{x}, \mathbf{y}\rangle = \langle \mathbf{x}, A\mathbf{y}\rangle$ for all $\mathbf{x}, \mathbf{y} \in \mathbb{C}^n$.

   (e) (5%) Please show that both BB^H and $B^H B$ are nonnegative definite for any complex matrix B.