EE205003 Linear Algebra, 2020 Fall Semester

Quiz # 10

DATE: Dec. 9th, 2020

1. (10%) Suppose A is a 7×6 matrix and all solutions of $A\mathbf{x} = \mathbf{0}$ are multiples of one nonzero vector. Will the equation $A\mathbf{x} = \mathbf{b}$ be solvable for all $\mathbf{b}$? Please give reason(s), otherwise no credits.

2. (15%) Let A be an $m \times n$ matrix. Please show that the linear system $A\mathbf{x} = \mathbf{b}$ is consistent for all $\mathbf{b} \in \mathbb{R}^m$ if and only if $\text{Ker}(A^T) = \{\mathbf{0}\}$.

3. (10%) Let $B = \{\mathbf{e}_1, \mathbf{e}_2\}$ be the standard basis and $C = \{(2, -2), (1, 5)\}$ another basis for $\mathbb{R}^2$. Please find the transition matrix for changing B-coordinates to C-coordinates.

4. (15%) Let B, C, D be three bases for a finite dimensional vector space V. Please show that if P is the transition matrix for changing B-coordinates to C-coordinates, and if Q is the transition matrix for changing C-coordinates to D-coordinates, then QP is the transition matrix for changing B-coordinates to D-coordinates.

5. (15%) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ with $T(\mathbf{x}) = A\mathbf{x}$, where $A = \begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix}$. Please find the matrix representation of T relative to the basis $B = \{(1, -1), (2, 3)\}$ for $\mathbb{R}^2$.

6. (20%) Find necessary and sufficient conditions on general 2×2 matrices A and B such that they are similar.

7. (15%) Is it true that every 2×2 real matrix is similar to its transpose? Please show it or give a counterexample, otherwise no credits.