EE205003 Linear Algebra, 2020 Fall Semester

Quiz # 9

DATE: Dec. 2nd, 2020

1. (15%) Consider

$$S = \{(3, 0, -2, 4, 1), (4, -2, -5, 2, 1), (-1, 1, -2, 0, 1),$$
$$(4, -3, -10, 0, 2), (2, -3, -1, -2, -1)\}$$

   in $\mathbb{R}^5$. Please find a basis for Span(S).

2. (15%) Please extend the set $S = \{(-1, 2, -3)\}$ to a basis for $\mathbb{R}^3$.

3. Let $L : \mathbb{P}_5 \to \mathbb{P}_5$ be defined by $L(p) = p'' - p'$. What are (8%) Ker(L), (7%) Range(L), and (5%) Dim(Codomain(L)) ? Please give reasons, otherwise no credits.

4. Let $\mathbb{R}^\infty$ be the vector space of all infinite sequences of the form $(x_1, x_2, \ldots)$ where x_i are arbitrary real numbers. Let U be the subset of $\mathbb{R}^\infty$ consisting of all sequences whose components satisfy $x_n = x_{n-1} - 2x_{n-2} - x_{n-3}$ for all $n \geq 4$. Please (8%) show that U is a subspace of $\mathbb{R}^\infty$ and (7%) find Dim(U).

5. (15%) Let B be an $n \times n$ non-invertible matrix. Let V be the set of all $n \times n$ matrices A such that $AB = \mathbf{0}$. Please show that V is a subspace of $\mathcal{M}_{n \times n}$ and find Dim(V).

6. (20%) Let A be an $m \times n$ matrix. Please show that Ker(A) $\cap$ Col(A^T) $= \{\mathbf{0}\}$.