EE205003 Linear Algebra, 2020 Fall Semester

Quiz # 8

DATE: November 25th, 2019

1. Please answer the following questions. You should give reason(s), otherwise no credits.

   (a) (10%) Is the set of all invertible 2×2 matrices a vector subspace of $\mathbb{R}^{2\times 2}$?

   (b) (10%) Is the set of all 2×2 matrices of the form $\begin{bmatrix} -a & a-b \\ b & a-c \end{bmatrix}$ a vector subspace of $\mathbb{R}^{2\times 2}$?

2. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ by $T((x_1, x_2, x_3)) = (x_1 - x_2 - 2x_3, -2x_1 + x_3)$.

   (a) (10%) Please find $T(U)$ with $U = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid -x_1+2x_2-x_3 = 0\}$.

   (b) (10%) Please find $T^{-1}(V)$ with $V = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1 - x_2 = 0\}$.

3. (15%) Do these two matrices have the same row space? Please give reason(s), otherwise no credits.

$$\begin{bmatrix} 1 & -3 & -2 & 2 \\ 0 & 1 & 2 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}, \quad \begin{bmatrix} -1 & 0 & -2 & 2 \\ 2 & 1 & 0 & -1 \\ 0 & -1 & 2 & 1 \end{bmatrix}.$$

4. (15%) Let $\mathbb{R}^\infty$ be the vector space of all infinite sequences of the form $(x_1, x_2, \ldots)$ where x_i are arbitrary real numbers. Let U be the subset of $\mathbb{R}^\infty$ consisting of all sequences that have only finitely many nonzero terms. Please show that U is a subspace of $\mathbb{R}^\infty$.

5. Let A and B be arbitrary matrices subject only to the condition that the product AB exists.

   (a) (10%) Consider these two inclusion relations: $\mathrm{Col}(AB) \subseteq \mathrm{Col}(A)$ and $\mathrm{Col}(AB) \subseteq \mathrm{Col}(B)$. Select the one of these that is always correct and prove it.

   (b) (5%) Under what condition(s), the selected inclusion relation in above becomes an equality relation.

6. (15%) Please show that the span of a nonempty set in a vector space is the smallest subspace containing that set. (Hint: A subspace W of a vector space V is called the smallest subspace containing a nonempty subset X of V if $X \subseteq W$ and for any subspace U of V such that $X \subseteq U$, we have $W \subseteq U$.)