# EE205003 Linear Algebra, 2020 Fall Semester

## Quiz # 7

DATE: November 18th, 2020

1. (15%) Consider three matrices

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ -5 & 1 & 0 & 0 & 0 \\ 12 & 120 & 3 & 0 & 0 \\ 97 & 18 & -3 & -2 & 0 \\ 0 & 110 & 0 & 23 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 31 & -221 & 219 & 414 \\ 0 & -2 & 101 & -13 & 39 \\ 0 & 0 & 1 & 401 & 22 \\ 0 & 0 & 0 & 3 & 32 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}, C = \begin{bmatrix} -2 & 0 & 0 & 0 & 0 \\ -3 & 4 & 0 & 0 & 0 \\ 12 & 0 & 1 & 0 & 0 \\ 87 & -18 & -3 & -2 & 0 \\ 13 & 10 & -1 & 3 & -1 \end{bmatrix}.$$

   Please compute $\mathrm{Det}(ABC)$. There will be no partial credits if you get a wrong answer.

2. (15%) Please calculate the determinant of the matrix

$$\begin{bmatrix} 0 & 3 & -2 & -2 \\ -1 & 1 & 0 & -3 \\ 2 & 3 & -2 & -3 \\ 3 & -1 & 0 & 2 \end{bmatrix}.$$

   There will be no partial credits if you get a wrong answer.

3. True or false. Please give a proof or a counterexample, otherwise no credit.
   (a) (10%) If $A$ is an invertible $n \times n$ matrix, then $\mathrm{Det}(A^{-2}) = (\mathrm{Det}(A))^{-2}$.
   (b) (10%) For any two $n \times n$ matrices $A, B$, we have $\mathrm{Det}(AB^T) = \mathrm{Det}(BA)$.
   (c) (10%) For any two $n \times n$ matrices $A, B$, we have $\mathrm{Det}(A - B) = \mathrm{Det}(A) - \mathrm{Det}(B)$.
   (d) (10%) If we change one entry in an $n \times n$ matrix $A$, then the determinant necessarily changes.

4. (15%) Please show that in each row of an $n \times n$ invertible matrix, there is an entry that can be changed to make the matrix non-invertible.

5. (15%) If the entries of a square matrix $A$ are integers, please show that $\mathrm{Det}(A)$ is an integer.