EE205003 Linear Algebra, 2020 Fall Semester

Quiz # 5

DATE: Oct. 21, 2020

1. Consider a linear transformation $T(\mathbf{x}) = A\mathbf{x}$ with

$$A = \begin{bmatrix} 1 & -1 & 0 & 1 \\ -2 & 1 & -3 & 1 \\ 0 & -1 & -3 & 3 \end{bmatrix}.$$

   (a) (4%) Please find the domain and co-domain of T.

   (b) (4%) Please fine the kernel of T.

   (c) (4%) Please fine the range of T.

   (d) (4%) Please determine whether T is injective.

   (e) (4%) Please determine whether T is surjective.

2. (15%) Determine whether there is a linear transformation T such that $T(-1,1) = (-1,2,-2)$, $T(0,1) = (0,1,-2)$, and $T(-3,1) = (2,-1,0)$.

3. (15%) Let T be a function from $\mathbb{R}^n$ to $\mathbb{R}^m$. Please show that T is a linear transformation if and only if $T(\alpha\mathbf{x} - \mathbf{y}) = \alpha T(\mathbf{x}) - T(\mathbf{y})$ for all scalars α and all vectors $\mathbf{x}, \mathbf{y}$ in $\mathbb{R}^n$.

4. (15%) Let

$$A = \begin{bmatrix} a & 0 \\ c & d \\ -1 & f \end{bmatrix}.$$

   Find values of a, c, d, f so that the linear transformation $\mathbf{x} \mapsto T(\mathbf{x}) = A\mathbf{x}$ is a one-to-one function from $\mathbb{R}^2$ to $\mathbb{R}^3$.

5. (15%) Please show that if T is a linear transformation from $\mathbb{R}^n$ onto $\mathbb{R}^m$, then $m \leq n$.

6. (10%) Please show that a linear transformation will map a finite linearly dependent indexed set into a linearly dependent indexed set.

7. (10%) Will a linear transformation map a finite linearly independent indexed set into a linearly independent indexed set? Please give reasons, otherwise no credits