EE205003 Linear Algebra, 2020 Fall Semester

Quiz # 4

DATE: Oct. 14th, 2020

1. (10%) Let U and V be two subsets of $\mathbb{R}^n$. Please show that if $U \subseteq \text{Span}(V)$, then $\text{Span}(U) \subseteq \text{Span}(V)$.

2. (15%) Please show that for any subset U in $\mathbb{R}^n$, we have $\text{Span}(\text{Span}(U)) = \text{Span}(U)$. (Hint: You may use the result in Problem 1 in above.)

3. Answer and explain these questions:

   (a) (5%) Can the span of a set be the empty set?

   (b) (5%) Can the span of a set contain one and only one vector?

   (c) (5%) If $\text{Span}(S) \subseteq \text{Span}(T)$, does it follow that $S \subseteq T$?

   (d) (5%) If $S \subseteq T$, does it follow that $\text{Span}(S) \subseteq \text{Span}(T)$?

4. (10%) Determine the values of $a, b,$ and c so that the vector (a, b, c) is not in the span of the set of three vectors $\{(1,3,1), (-1,-2,2), (-1,-1,5)\}$.

5. (15%) A plane in $\mathbf{R}^4$ is represented parametrically by the equation: $\mathbf{x} = \mathbf{w} + s\mathbf{u} + t\mathbf{v}$, where $\mathbf{w} = (-1, 2, 0, -1), \mathbf{u} = (1, 1, -2, -1), \mathbf{v} = (1, 0, -1, 0)$. Please represent this plane by a linear system.

6. Let H be an affine space in $\mathbb{R}^5$ represented by a linear system $A\mathbf{x} = \mathbf{b}$, where

$$[A|\mathbf{b}] = \left[\begin{array}{ccccc|c} -2 & 1 & 0 & 0 & -1 & -1 \\ 1 & -2 & -2 & -1 & 0 & 0 \end{array}\right].$$

   (a) (12%) Find a parametric representation of H.

   (b) (3%) What is the dimension of H ?

7. (15%) Please show Theorem 2.2.A: A k-dimensional affine space H in $\mathbb{R}^n$ can be represented by a system of $(n-k)$ linear equations with n unknowns.