EE205003 Linear Algebra, 2020 Fall Semester

Quiz # 3

DATE: Oct. 7th, 2020

1. (15%) Consider a matrix $A = \begin{bmatrix} -1 & 2 & 0 & 1 \\ 2 & -1 & 3 & -8 \\ 1 & -1 & 1 & -3 \end{bmatrix}$. Find its rank and find a set of vectors whose span is its kernel.

2. (15%) Consider a square matrix $A = \begin{bmatrix} -1 & 2 & -1 \\ 2 & 0 & 3 \\ 1 & -1 & 2 \end{bmatrix}$. Is A invertible ? If yes, find its inverse A^{-1}. If no, explain the reason(s). (Hint: Solve the system $AX = I_{3\times 3}$, where X is the unknown 3×3 matrix and $I_{3\times 3}$ is the 3×3 identity matrix.)

3. (15%) Consider a set of vectors $\{(1, 0, -1, 2), (3, 0, 1, 4), (0, 0, -2, 1)\}$. Is it linearly dependent? If yes, find a nontrivial linear relation for this set of vectors. If no, explain the reason(s).

4. (15%) Please show that if $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent, then $\{\mathbf{v}_1 - \mathbf{v}_2 + \mathbf{v}_3, -2\mathbf{v}_2, \mathbf{v}_1 - \mathbf{v}_3\}$ is also linearly independent.

5. (10%) What are the ranks of the matrices in this infinite sequence? $A_1 = [1]$, $A_2 = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$, $A_3 = \begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 7 & 8 & 9 \end{bmatrix}$, $A_4 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 8 & 7 & 6 & 5 \\ 9 & 10 & 11 & 12 \\ 16 & 15 & 14 & 13 \end{bmatrix}$ and so on. Please give reason(s).

6. (15%) If two matrices are row equivalent to each other, do they have the same rank? If two matrices have the same rank, are they row equivalent to each other? You must give reason(s), otherwise no credit.

7. (15%) Please show Theorem 1.3.13: The column vectors of a matrix form a linearly independent set if and only if there is a pivot position in each column of the matrix.