EE205003 Linear Algebra, 2020 Fall Semester

Quiz # 2

DATE: Sept. 30, 2020

1. (10%) What is the column space $\mathrm{Col}(A)$ of the matrix $A = \begin{bmatrix} 0 & 1 & -1 & 8 \\ -1 & -2 & 0 & 1 \\ 2 & 0 & -2 & -3 \end{bmatrix}$.

2. (15%) Find the general solution of a linear system with augmented matrix

$$\left[\begin{array}{ccccc|c} 0 & 1 & 3 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & -3 \\ 0 & 0 & 0 & 0 & 2 & 2 \end{array}\right].$$

3. (15%) Under what conditions on a, b, c, d does the augmented matrix correspond a consistent linear system: $\left[\begin{array}{cccc|c} -1 & 0 & 3 & -9 & a \\ 2 & -3 & -3 & -3 & b \\ 0 & 1 & -1 & 7 & c \\ -1 & 2 & 1 & 5 & d \end{array}\right]$?

4. (10%) Let $A = [\mathbf{a}_1 \mathbf{a}_2 \dots \mathbf{a}_n]$ and $B = [\mathbf{b}_1 \mathbf{b}_2 \dots \mathbf{b}_n]$ be two row equivalent $m \times n$ matrices. For $1 \leq k \leq n$, let $A_k = [\mathbf{a}_1 \mathbf{a}_2 \dots \mathbf{a}_k]$ and $B_k = [\mathbf{b}_1 \mathbf{b}_2 \dots \mathbf{b}_k]$ be submatrices of A and B respectively. Please show that A_k and B_k are also row equivalent.

5. (10%) Under what circumstances is this implication not true? [(P or Q) and (P or R)] $\Rightarrow$ (Q or R).

6. (10%) If $P \Rightarrow Q$, does it follows that P is true whenever Q is true? Provide the truth table for $(P \Rightarrow Q) \Rightarrow (Q \Rightarrow P)$.

7. (15%) What is the negative (denial) of the assertion that there exists an $\epsilon > 0$, for all positive integer m such that there exists an integer $n \geq m$, $|P(n) - P(m)| < \epsilon$.

8. (15%) Is the assertion $P \Rightarrow Q$ equivalent to the assertion $\neg Q \Rightarrow \neg P$? Provide the truth table for $(P \Rightarrow Q) \Rightarrow (\neg Q \Rightarrow \neg P)$ and the truth table for $(\neg Q \Rightarrow \neg P) \Rightarrow (P \Rightarrow Q)$.