EECS205003 Linear Algebra, Fall 2020

Quiz # 2, Solutions

Prob. 1:

$$\mathrm{Col}(A) = \left\{ \alpha_1 \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} + \alpha_2 \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + \alpha_3 \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix} + \alpha_4 \begin{bmatrix} 8 \\ 1 \\ -3 \end{bmatrix} \middle| \alpha_1, \alpha_2, \alpha_3 \text{ and } \alpha_4 \in \mathbb{R} \right\}.$$

Prob. 2: By elementary row operations, we have

$$\left[\begin{array}{ccccc|c} 0 & 1 & 3 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & -3 \\ 0 & 0 & 0 & 0 & 2 & 2 \end{array}\right] \to \left[\begin{array}{ccccc|c} 0 & 1 & 3 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right].$$

We can rewrite the linear system as

$$\begin{bmatrix} 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}.$$

We choose x_1, x_3 as free variables. Then x_2, x_4, x_5 are dependent variables and become

$$\begin{cases} x_2 = & -3x_3 + & 1, \\ x_4 = & & 3, \\ x_5 = & & 1. \end{cases}$$

The general solution of the linear system is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 3 \\ 1 \end{bmatrix}.$$

Prob. 3:

Note that

$$\left[\begin{array}{cccc|c} -1 & 0 & 3 & -9 & a \\ 2 & -3 & -3 & -3 & b \\ 0 & 1 & -1 & 7 & c \\ -1 & 2 & 1 & 5 & d \end{array}\right] \to \left[\begin{array}{cccc|c} -1 & 0 & 3 & -9 & a \\ 0 & -3 & 3 & -21 & b+2a \\ 0 & 1 & -1 & 7 & c \\ 0 & 2 & -2 & 14 & d-a \end{array}\right] \to \left[\begin{array}{cccc|c} -1 & 0 & 3 & -9 & a \\ 0 & 0 & 0 & 0 & b+2a+3c \\ 0 & 1 & -1 & 7 & c \\ 0 & 0 & 0 & 0 & d-a-2c \end{array}\right].$$

If $b + 2a + 3c = 0$ and $d - a - 2c = 0$, then this linear system is consistent.

Prob. 4:

Since $\mathbf{A}$ and $\mathbf{B}$ are row equivalent, then there exists a sequence $\mathbf{F}_1, \mathbf{F}_2, ..., \mathbf{F}_s$ of elementary $(\mathbf{E}_{ij}(\alpha), \alpha \in \mathbb{R})$ or scaling $(\mathbf{S}_i(\alpha), \alpha \in \mathbb{R}\backslash\{0\})$ or permutation $(\mathbf{P}_{ij})$ $m \times m$ matrices such that

$$\mathbf{B} = \mathbf{F}_s...\mathbf{F}_2\mathbf{F}_1\mathbf{A}$$

where for each column of $\mathbf{B}$, $\mathbf{b}_i = \mathbf{F}_s...\mathbf{F}_2\mathbf{F}_1\mathbf{a}_i$, $i = 1, 2, , ..., n$. So $\mathbf{B}_k = \mathbf{F}_s...\mathbf{F}_2\mathbf{F}_1\mathbf{A}_k$, $\mathbf{A}_k$ and $\mathbf{B}_k$ are row equivalent.

Prob. 5:

Look at the following truth table. It is clear that the implication is not true when P is T, Q is F, and R is F.

P	Q	R	(P or Q)	(P or R)	[(P or Q) and (P or R)]	(Q or R)
F	F	F	F	F	F	F
F	F	T	F	T	F	T
F	T	F	T	F	F	T
F	T	T	T	T	T	T
T	F	F	T	T	T	F
T	F	T	T	T	T	T
T	T	F	T	T	T	T
T	T	T	T	T	T	T

Prob. 6:

No.

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$(P \Rightarrow Q) \Rightarrow (Q \Rightarrow P)$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	T
T	T	T	T	T

Prob. 7:

$$\forall \epsilon > 0, \exists \text{ a positive integer } m \text{ s.t. } \forall \text{ integer } n \geq m, |P(n) - P(m)| \geq \epsilon$$

Prob. 8:

Yes.

P	Q	$P \Rightarrow Q$	$\neg Q \Rightarrow \neg P$	$(P \Rightarrow Q) \Rightarrow (\neg Q \Rightarrow \neg P)$	$(\neg Q \Rightarrow \neg P) \Rightarrow (P \Rightarrow Q)$
F	F	T	T	T	T
F	T	T	T	T	T
T	F	F	F	T	T
T	T	T	T	T	T