EE205003 Linear Algebra, 2020 Fall Semester

Quiz # 1

DATE: Sept. 23, 2020

1. (15%) Find two row echelon forms of the matrix $\begin{bmatrix} -2 & 0 & -1 & 2 \\ 0 & 3 & -1 & 2 \\ 4 & 3 & 2 & 4 \\ 2 & 3 & 1 & 3 \\ 3 & 0 & 1 & -1 \end{bmatrix}$.

2. (15%) Find the reduce row echelon form of the matrix $\begin{bmatrix} 0 & 2 & 2 & 0 & 0 \\ -3 & -1 & -2 & -1 & 1 \\ 3 & 1 & 0 & -1 & 0 \\ -3 & -1 & 0 & 1 & 1 \end{bmatrix}$.

3. (15%) Establish that if a matrix has all integer entries, then it is row equivalent to a matrix in row echelon form having only integer entries. (5%) Can we make the same assertion for the reduced row echelon form? Why?

4. (15%) Let $f : A \to B$ be a function from a set A to a set B. Let E be a subset of A. Define $f(E) = \{f(a) \in B \mid a \in E\}$. Let $\{E_i, i \in I\}$ be a collection of subsets of A indexed by I. Find the relationship between $f(\bigcap_{i\in I} A_i)$ and $\bigcap_{i\in I} f(A_i)$. Establish your answer with suitable deductions and give examples to convince one that no stronger relationship is valid in general.

5. (10%) If P implies Q, does it follow that not-P implies not-Q? Give the truth table for the latter.

6. (15%) Use induction to prove that $n^2 + n + 1$ is odd for all $n \in N$.

7. A real number is said to be **rational** if it can be expressed as the quotient of two integers. In the contrary case, the number is said to be **irrational**. Substantiate the correct statements and give suitable counterexamples for the incorrect ones:

   a. (5%) The sum of any two rational numbers is rational.

   b. (5%) The sum of a rational number and an irrational number is irrational.