

## 2018 Fall EECS205003 Linear Algebra - Quiz 8

Name:

ID:

$$1. P^{-1}AP = D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

$$(P^{-1}AP)^T = D^T$$

$$P^T A^T (P^T)^{-1} = D^T = D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$Q^{-1}A^T Q = D$$

$$Q = (P^T)^{-1} = [w_1, w_2, w_3] = \begin{bmatrix} 3 & -5 & 3 \\ -1 & 3 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A^T w_2 = \lambda_2 w_2 = w_2 = c \begin{bmatrix} -5 \\ 3 \\ 1 \end{bmatrix} = c' \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

$$2. \det \begin{bmatrix} 1 - \lambda & -12 \\ -12 & -6 - \lambda \end{bmatrix} = (1 - \lambda)(-6 - \lambda) - 12^2 = \lambda^2 + 5\lambda - 150 = 0$$

$$\lambda = 10, -15$$

$$\lambda = 10 \rightarrow \begin{bmatrix} -9 & -12 \\ -12 & -16 \end{bmatrix} x = 0 \rightarrow x_1 = c_1 \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

$$\lambda = -15 \rightarrow \begin{bmatrix} 16 & -12 \\ -12 & 9 \end{bmatrix} x = 0 \rightarrow x_2 = c_2 \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$P = [v_1, v_2] = \begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix}$$

$$P^{-1}AP = D = \begin{bmatrix} 10 & 0 \\ 0 & -15 \end{bmatrix}$$