

2018 Fall EECS205003 Linear Algebra - Quiz 7

Name:

ID:

1. (a) $S = \text{Span}[v_1, v_2, v_3]$

$$u_1 = v_1 = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$

$$u_2 = v_2 - \frac{\langle v_2, u_1 \rangle}{\|u_1\|^2} u_1 = v_2 - \frac{6}{12} u_1 = \frac{1}{2} \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

$$u_3 = v_3 - \frac{\langle v_3, u_1 \rangle}{\|u_1\|^2} u_1 - \frac{\langle v_3, u_2 \rangle}{\|u_2\|^2} u_2 = v_3 - \frac{18}{12} u_1 - \frac{-1}{1} u_2 = \begin{bmatrix} -2 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\phi_1 = \frac{1}{\sqrt{12}} \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix}, \phi_2 = \frac{1}{2} \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \phi_3 = \frac{1}{\sqrt{6}} \begin{bmatrix} -2 \\ 0 \\ 1 \\ 1 \end{bmatrix},$$

- (b) $\text{Proj}_S(b) = \langle b, \phi_1 \rangle \phi_1 + \langle b, \phi_2 \rangle \phi_2 + \langle b, \phi_3 \rangle \phi_3$

$$= \frac{7}{12} \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix} + \frac{11}{4} \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix} + \frac{-1}{3} \begin{bmatrix} -2 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 3 \\ 3 \end{bmatrix} = p$$