2018 Fall EECS205003 Linear Algebra - Quiz 6

Name: ID:

1. You are analysing parameters of your game character. Suppose one parameter strength s is controlled by another three parameters (a_1, a_2) for some (w_0, w_1, w_2) so that

$$s = w_0 + w_1 a_1 + w_2 a_2$$

You have collected some data s = 4, 8, 3, 5, 2 and 6 corresponding to (a_1, a_2) = (-1, 1), (3, -4), (-2, 2), (0, -2), (-1, -2) and (1, -1). Please answer the following questions.

(a) Try to write the problem into matrix form $A\mathbf{w} = \mathbf{s}$ where $\mathbf{w} = \begin{bmatrix} w_0 & w_1 & w_2 \end{bmatrix}^\intercal$.

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 3 & -4 \\ 1 & -2 & 2 \\ 1 & 0 & -2 \\ 1 & -1 & -2 \\ 1 & 1 & -1 \end{bmatrix}, \mathbf{s} = \begin{bmatrix} 4 \\ 8 \\ 3 \\ 5 \\ 2 \\ 6 \end{bmatrix}$$

(b) Find $\widehat{\mathbf{w}}$ so that $\widehat{\mathbf{s}} = A\widehat{\mathbf{w}}$ is closest to $\mathbf{s}$

$$A^\intercal A = \begin{bmatrix} 6 & 0 & -6 \\ 0 & 16 & -16 \\ -6 & -16 & 30 \end{bmatrix}, A^\intercal \mathbf{s} = \begin{bmatrix} 28 \\ 18 \\ -42 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 6 & 0 & -6 & 28 \\ 0 & 16 & -16 & 18 \\ -6 & -16 & 30 & -42 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 6 & 0 & -6 & 28 \\ 0 & 16 & -16 & 18 \\ 0 & 0 & 8 & 4 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{31}{6} \\ 0 & 1 & 0 & \frac{13}{8} \\ 0 & 0 & 1 & \frac{1}{2} \end{array}\right]$$

$$\Rightarrow \widehat{\mathbf{w}} = (A^\intercal A)^{-1} A^\intercal \mathbf{s} = \begin{bmatrix} \frac{31}{6} \\ \frac{13}{8} \\ \frac{1}{2} \end{bmatrix}$$