2018 Fall EE205003 Linear Algebra - Quiz 5 sol.

Name: ID:

1. Given a m by n matrix A has rank r. What is the relation between the rank r, m and n when the number of solutions to $A\mathbf{x} = \mathbf{b}$ is
   (a) 0 or 1, depending on $\mathbf{b}$.(Please explain the relation between $\mathbf{b}$ and A)
   (b) ∞, regardless of $\mathbf{b}$.
   (c) 0 or ∞, depending on $\mathbf{b}$.(Please explain the relation between $\mathbf{b}$ and A)
   (d) 1, regardless of $\mathbf{b}$.

Ans:

(a) $r = n$ and $n < m$
   (i) $\mathbf{b} \in Col(A) \leftrightarrow$ unique solution.
   (ii) $\mathbf{b} \notin Col(A) \leftrightarrow$ no solution.

(b) $r = m$ and $n > m$

(c) $r < n$ and $r < m$
   (i) $\mathbf{b} \in Col(A) \leftrightarrow$ Infinite solutions.
   (ii) $\mathbf{b} \notin Col(A) \leftrightarrow$ no solution.

(d) $r = n = m$