# 2018 Fall EECS205003 Linear Algebra - Quiz 2 Solution

Name: ID:

1. Let a set of linear equations be $\begin{cases} x_1 - x_2 + 3x_3 = -1 \\ 2x_1 + x_2 - x_3 = 1 \\ x_1 + 3x_2 + 2x_3 = 2 \end{cases}$

(1) If we write the linear equations into matrix form $A\mathbf{x} = \mathbf{b}$, where $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ what are $A$ and $\mathbf{b}$ ?

$A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & -1 \\ 1 & 3 & 2 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$

(2) Find out the first and the second pivot.

The first pivot we can find out directly from first equation $\Rightarrow 1$

Eliminate $A_{21}$ according to the first pivot

$\Rightarrow$ (row 2 of $A$) $-$ 2$\times$(row 1 of $A$)

$$\Rightarrow \begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & -1 \\ 1 & 3 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 3 \\ 0 & 3 & -7 \\ 1 & 3 & 2 \end{bmatrix}$$

Then we have the second pivot $\Rightarrow 3$

(3) Find out elimination matrices $E_{21}$, $E_{31}$ and $E_{32}$.

According to (2), the first pivot is 1

We eliminate $A_{21}$ via (row 2 of $A$) $-$ 2$\times$(row 1 of $A$)

$$\Rightarrow E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Similarly, we eliminate $A_{31}$ via (row 3 of $A$) $-$ 1$\times$(row 1 of $A$)

$$\Rightarrow E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

After elimination by the first pivot

$$\begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & -1 \\ 1 & 3 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 3 \\ 0 & 3 & -7 \\ 1 & 3 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 3 \\ 0 & 3 & -7 \\ 0 & 4 & -1 \end{bmatrix} \overset{\text{let}}{=} A'$$

We eliminate $A'_{32}$ by the second pivot 3

$\Rightarrow$ (row 3 of $A'$) $- \frac{4}{3}\times$(row 2 of $A'$)

$$\Rightarrow E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{4}{3} & 1 \end{bmatrix}$$

And we can get the triangular matrix $\begin{bmatrix} 1 & -1 & 3 \\ 0 & 3 & -7 \\ 0 & 0 & \frac{25}{3} \end{bmatrix}$