2018 Fall EECS205003 Linear Algebra - Midterm II

1. (12%) Suppose $A = \begin{bmatrix} 2 & 4 & 3 \\ 2 & 5 & 7 \\ 4 & 9 & 10 \end{bmatrix}$, please answer the following questions:
   (a) (4%) Show whether the columns of A are linear independent or not.
   (b) (4%) Find a basis of the orthogonal subspace of $C(A)$.
   (c) (4%) Find a basis of the orthogonal subspace of $C(A^T)$.

2. (17%) Please answer the following questions:
   (a) (4%) Find the projection $\mathbf{p}$ of the vector $\mathbf{b} = (1, 2, 6)$ onto the plane $x + y + z = 0$ in $\mathbf{R^3}$.
   (b) (4%) Find the line which best fits the four data points $(1, 2), (2, 1), (3, 3)$ and $(4, 2)$ in the sense of least squares.
   (c) (4%) Let $S = \{\mathbf{v_1}, \mathbf{v_2}\}$ be the set of the following vectors in $\mathbf{R^4}$:

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}.$$

   Find the orthogonal basis of the subspace span(S) of $\mathbf{R^4}$.
   (d) (5%) Use Gram Schmidt process to find the QR factorization of $A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \\ 0 & 3 \end{bmatrix}$.

3. (16%) Let $\mathbf{u_1} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{u_2} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \mathbf{u_3} = \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \mathbf{u_4} = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

   $U_{4\times4} = \begin{bmatrix} \mathbf{u_1} & \mathbf{u_2} & \mathbf{u_3} & \mathbf{u_4} \end{bmatrix}$.
   (a) (4%) Is $U_{4\times4}$ an orthogonal matrix? Please give your reason.
   (b) (4%) Find U^{-1}.
   (c) (4%) Suppose that $\mathbf{y} = c_1\mathbf{u_1} + c_2\mathbf{u_2} + c_3\mathbf{u_3} + c_4\mathbf{u_4}$. Find c_3 and c_4.
   (d) (4%) Let $W = \text{Span}\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$. Find the vector in W that is closest to $\mathbf{y}$.

4. (13%) Consider the 4 by 4 matrices:

$$A = \begin{bmatrix} 2a & c & b & 0 \\ b & a+d & 0 & b \\ c & 0 & a+d & c \\ 0 & c & b & 2d \end{bmatrix}, B = \begin{bmatrix} 1 & 101 & 201 & 301 \\ 2 & 102 & 202 & 302 \\ 3 & 103 & 203 & 303 \\ 4 & 104 & 204 & 304 \end{bmatrix}$$

   (a) (5%) Find determinant of A. (In terms of $a + d$ and $ad - bc$).
   (b) (4%) In this question, please find the relation between a, b, c, d, such that A has no inverse.
   (c) (4%) Find determinant of AB.

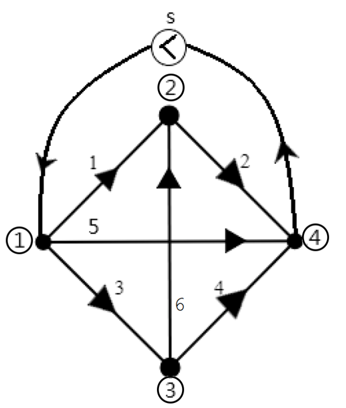
Please turn over to continue.

5. (10%) Let $M = \begin{bmatrix} a & b & 0 & 0 \\ 0 & 0 & a & b \\ c & d & 0 & 0 \\ 0 & 0 & c & d \end{bmatrix}$, where $a > b > 0$ and $d > c > 0$,

(a) (5%) Find the determinant of the matrix M

(b) (5%) Find M^{-1}

6. (17%) The following figure shows electrical network **G**. The conductance $(c_1, c_2, c_3, c_4, c_5, c_6) = (1, \frac{1}{2}, 1, \frac{1}{2}, 1, \frac{1}{4})$ to corresponding edges. And the current source s flows into node 1 and flows out of node 4. Please answer the following questions:



(a) (7%) Represent the incident matrix A and try to use loops in **G** to indicate a basis of the left nullspace of A instead of computing elimination.

(b) (10%) Suppose that potential at node 1 is v. Find potential at each node and current on each edge.