2018 Fall EECS205003 Linear Algebra - Homework 4

Name: ID:

1. (a) Find the basis for the subspace S in R^4 spanned by all solutions of $x_1 + x_2 + x_3 - x_4 = 0$.

   (b) Find a basis for the orthogonal complement $S^{\perp}$.

   (c) Find b_1 in S and b_2 in $S^{\perp}$ so that $b_1 + b_2 = b = (1, 1, 1, 1)$.

2. If A is m *by* n and B is n *by* m, block multiplication gives $detM = detAB$:

   $$M = \begin{bmatrix} 0 & A \\ -B & I \end{bmatrix} = \begin{bmatrix} AB & A \\ 0 & I \end{bmatrix} \begin{bmatrix} I & 0 \\ -B & I \end{bmatrix}$$

   If A is a single row and B is a single column what is $detM$? If A is a column and B is a row what is $detM$? Do a 3 *by* 3 example of each.

3. Let $b = C + Dt$ be closest line to the points $(b, t) = (0, 0), (8, 1), (8, 3)$, and $(20, 4)$. Find the least squares solution $\hat{\mathbf{x}} = (C, D)$.

4. Find the determinants of rotations and reflections:

   $$\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \text{ and } \begin{bmatrix} 1 - 2cos^2\theta & -2cos\theta sin\theta \\ -2cos\theta sin\theta & 1 - 2sin^2\theta \end{bmatrix}$$

5. Given five sample points $(t,\ b) = (0, 0), (2, 1), (-1, -3), (1, 2)$ and $(-2, 1)$. Please answer the following questions.

   (a) Solve the closest line to sample points via least square approximation.

   (b) If we want to fetch a closest parabola $C + Dt + Et^2$ instead of a line, try to solve it by least square approximation.

   (c) Compare the error E between line and parabola, describe what you haved discovered. If we fit a higher order polynomial to sample points, guess what will happen to error E according to you discovery.

6. Use row operations to show that the 3 by 3 "Vandermonde determinant" is

(a) $\det \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix} = (b-a)(c-a)(c-b)$

(b) $\det \begin{bmatrix} 1 & a & a^2 \\ 1 & a & 1 \\ a^2 & a & 1 \end{bmatrix} = (1-a^2)^2$