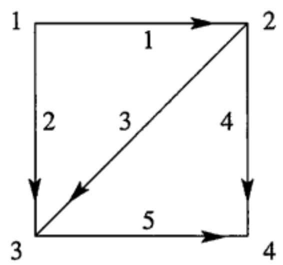
2018 Fall EECS205003 Linear Algebra - Homework 3

Name: ID:

1. With conductances $c_1 = c_2 = 2$ and $c_3 = c_4 = c_5 = 3$, multiply the matrices $A^T CA$.
   Find a solution to $A^T CAx = f = (1, 0, 0, -1)$.
   Write these potentials x and currents $y = -CAx$ on the nodes and edges of the square graph.



2. (a) Find the condition on (b_1, b_2, b_3) for $A\mathbf{x} = \mathbf{b}$ to be solvable.

$$A = \begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

   (b) If $\mathbf{b} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$, find the complete solution.

3. Decide the dependence or independence of
   (a) the vectors (1, 3, 2) and (2, 2, 3) and (3, 3, 5)
   (b) the vectors (1, -3, 2) and (2, 1, -3) and (-3, 2, 1).

4. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$. Find bases of four subspaces without computing A.

5. Assume two $m \times n$ matrices $A = \begin{bmatrix} I & F_A \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} I & F_B \\ 0 & 0 \end{bmatrix}$ have the same four subspaces.
   Prove $F_A = F_B$.

6. **M** is the space of 3 by 3 matrices. Multiply every matrix X in **M** by

$$\begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

(a) Which matrices X lead to $AX =$ zero matrix? And, what is the dimension of the space X spans?

(b) Which matrices have the form AX for some matrix X ? Also, what is the dimension of the space it spans?