

2018 Fall EECS205003 Linear Algebra - Homework 2 sol.

Name:

ID:

1. $PA = LU$ is $\begin{bmatrix} 1 & & 1 \\ & 1 & \\ 1 & & \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 0 & 3 & 8 \\ 2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & & \\ 0 & 1 & \\ 0 & 1/3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ & 3 & 8 \\ & & -2/3 \end{bmatrix}$.

If we wait to exchange and a_{12} is the pivot, $A = L_1 P_1 U_1 = \begin{bmatrix} 1 & & \\ 3 & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} & 1 & \\ & & 1 \\ 1 & & \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ do not have $(1, 1, 1)$ in $\mathbf{C}(A)$.

$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 3 & 6 & 0 \end{bmatrix}$ has $\mathbf{C}(A)$ as a line.

3.

4.

5.

6. $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ has $A^2 = 0$. But the diagonal entries of $A^T A$ are dot products of columns of A with themselves. If $A^T A = 0$, zero dot products \Rightarrow zero columns $\Rightarrow A =$ zero matrix.

7. As picture:

$$\begin{aligned} & \left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & \textcircled{1} & -1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\times(-2)} \begin{array}{c} R_3 - 2 \times R_2 \rightarrow R_3 \\ \equiv \end{array} \left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & \textcircled{3} & 0 & -2 & 1 \end{array} \right) \xrightarrow{\times(1/3)} \begin{array}{c} R_3 / (3) \rightarrow R_3 \\ \equiv \end{array} \left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & \textcircled{1} & 0 & -2/3 & 1/3 \end{array} \right) \xrightarrow{\times(1)} \\ & \begin{array}{c} \dots \\ \equiv \end{array} \left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/3 & 1/3 \\ 0 & 0 & \textcircled{1} & 0 & -2/3 & 1/3 \end{array} \right) \xrightarrow{\times(-2)} \begin{array}{c} \dots \\ \equiv \end{array} \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 4/3 & -2/3 \\ 0 & \textcircled{1} & 0 & 0 & 1/3 & 1/3 \\ 0 & 0 & 1 & 0 & -2/3 & 1/3 \end{array} \right) \xrightarrow{\times(-2)} \begin{array}{c} R_1 - 2 \times R_2 \rightarrow R_1 \\ \equiv \end{array} \\ & \begin{array}{c} \dots \\ \equiv \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2/3 & -4/3 \\ 0 & 1 & 0 & 0 & 1/3 & 1/3 \\ 0 & 0 & 1 & 0 & -2/3 & 1/3 \end{array} \right) \\ & \begin{array}{c} \dots \\ \equiv \end{array} \left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 2/3 & -4/3 \\ 0 & 1 & -1 & 0 & 1/3 & 1/3 \\ 0 & 2 & 1 & 0 & -2/3 & 1/3 \end{array} \right) \xrightarrow{\times(-1)} \begin{array}{c} \dots \\ \equiv \end{array} \left(\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 2/3 & -4/3 \\ 0 & 1 & -1 & 0 & 1/3 & 1/3 \\ 0 & -2 & -1 & 0 & 2/3 & -1/3 \end{array} \right) \end{aligned}$$

$$8. U = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -5 & 5 \\ 0 & 0 & -2 \end{bmatrix}, L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & \frac{4}{5} & 1 \end{bmatrix}$$

$$L\mathbf{c} = \mathbf{b}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & \frac{4}{5} & 1 \end{bmatrix} \mathbf{c} = \begin{bmatrix} -1 \\ 8 \\ 5 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} -1 \\ 10 \\ 2 \end{bmatrix}$$

$$U\mathbf{x} = \mathbf{c}$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -5 & 5 \\ 0 & 0 & -2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} -1 \\ 10 \\ 2 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$