2018 Fall EECS205003 Linear Algebra - Homework 2 sol.

Name: ID:

1. $PA = LU$ is $\begin{bmatrix} & & 1 \\ & 1 & \\ 1 & & \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ 0 & 3 & 8 \\ 2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & & \\ 0 & 1 & \\ 0 & 1/3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 1 \\ & 3 & 8 \\ & & -2/3 \end{bmatrix}$.

   If we wait to exchange and a_{12} is the pivot, $A = L_1 P_1 U_1 = \begin{bmatrix} 1 & & \\ 3 & 1 & \\ & & 1 \end{bmatrix}\begin{bmatrix} & 1 & \\ & & 1 \\ 1 & & \end{bmatrix}\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ do not have $(1, 1, 1)$ in $\mathbf{C}(A)$.

   $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 3 & 6 & 0 \end{bmatrix}$ has $\mathbf{C}(A)$ as a line.

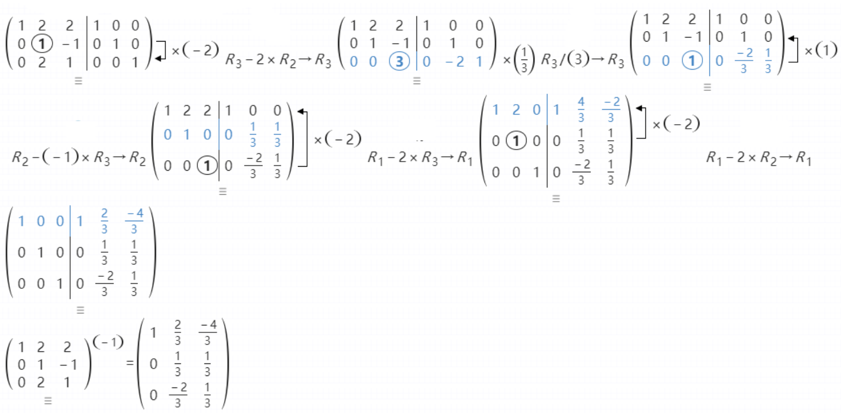
3.

4.

5.

6. $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ has $A^2 = 0$. But the diagonal entries of $A^T A$ are dot products of columns of A with themselves. If $A^T A = 0$, zero dot products $\Rightarrow$ zero columns $\Rightarrow$ $A =$ zero matrix.

7. As picture:



8. $U = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -5 & 5 \\ 0 & 0 & -2 \end{bmatrix}$, $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & \frac{4}{5} & 1 \end{bmatrix}$

$L\mathbf{c} = \mathbf{b}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & \frac{4}{5} & 1 \end{bmatrix} \mathbf{c} = \begin{bmatrix} -1 \\ 8 \\ 5 \end{bmatrix}, \ \mathbf{c} = \begin{bmatrix} -1 \\ 10 \\ 2 \end{bmatrix}$$

$U\mathbf{x} = \mathbf{c}$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -5 & 5 \\ 0 & 0 & -2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} -1 \\ 10 \\ 2 \end{bmatrix}, \ \mathbf{x} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$