2018 Fall EECS205003 Linear Algebra - Homework 2

Name: ID:

1. Factor the following matrix into $PA = LU$. Factor it also into $A = L_1 P_1 U_1$

   (hold the exchange of row 3 until 3 times row 1 is subracted from row 2):

   $$A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 3 & 8 \\ 2 & 1 & 1 \end{bmatrix}$$

2. Construct a 3×3 matrix whose column space contains (1,1,0) and (1,0,1) but not (1,1,1).

   Construct a 3 by 3 matrix whose column space is only a line.

3. Decide the dependence or independence of
   (a) the vectors (1, 3, 2) and (2, 2, 3) and (3, 3, 5)
   (b) the vectors (1, -3, 2) and (2, 1, -3) and (-3, 2, 1).

4. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$. Find bases of four subspaces without computing A.

5. Assume two $m \times n$ matrices $A = \begin{bmatrix} I & F_A \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} I & F_B \\ 0 & 0 \end{bmatrix}$ have the same four subspaces.
   Prove $F_A = F_B$.

6. Show that $A^2 = 0$ is possible but $A^T A = 0$ is not possible (unless $A =$ zero matrix).

7. Use Gauss-Jordan Elimination (session 7 p.6) to find K^{-1} (Please provide complete derivation to get full credit)

   $$K = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & -1 \\ 0 & 2 & 1 \end{bmatrix}$$

8. Find solution $\mathbf{x}$ of the following equations, by using $A = LU$, then solve $L\mathbf{c} = \mathbf{b}$ and $U\mathbf{x} = \mathbf{b}$ sequentially to get $\mathbf{x}$.

$$\begin{aligned} x + 2y - z &= -1 \\ 2x - y + 3z &= 8 \\ x - 2y + z &= 5 \end{aligned}$$