2018 Fall EECS205003 Linear Algebra - Homework 1 sol.

Name: ID:

1. Elimination produces the pivots $a,\ a-b,\ \&\ a-b$. $A^{-1} = \frac{1}{a(a-b)}\begin{bmatrix} a & 0 & -b \\ -a & a & 0 \\ 0 & -a & a \end{bmatrix}$

2. $\begin{bmatrix} I & 0 \\ -C & I \end{bmatrix}$ and $\begin{bmatrix} -D & I \\ I & 0 \end{bmatrix}$

3. (a) $AB = \begin{bmatrix} -1 \\ 4 \\ 7 \end{bmatrix}\begin{bmatrix} 1 & 3 & 9 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix}\begin{bmatrix} -3 & 2 & 6 \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \\ -4 \end{bmatrix}\begin{bmatrix} -2 & -8 & 1 \end{bmatrix}$
$= \begin{bmatrix} -1 & -3 & -9 \\ 4 & 12 & 36 \\ 7 & 21 & 63 \end{bmatrix} + \begin{bmatrix} -6 & 4 & 12 \\ -9 & 6 & 18 \\ 9 & -6 & -18 \end{bmatrix} + \begin{bmatrix} 4 & 16 & -2 \\ 0 & 0 & 0 \\ 8 & 32 & -4 \end{bmatrix}$
$= \begin{bmatrix} -3 & 17 & -1 \\ -5 & 18 & 54 \\ 24 & 47 & 41 \end{bmatrix}$

   (b) $AC = \begin{bmatrix} A\mathbf{c}_1 & A\mathbf{c}_2 & A\mathbf{c}_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

4. (a) $(A+B)^2 = \left(\begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}\right)^2 = \begin{bmatrix} 44 & 60 \\ 84 & 116 \end{bmatrix}$

   (b) $A^2 + 2AB + B^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + 2\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$
$= \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} + \begin{bmatrix} 20 & 26 \\ 44 & 58 \end{bmatrix} + \begin{bmatrix} 16 & 21 \\ 28 & 37 \end{bmatrix} = \begin{bmatrix} 43 & 57 \\ 87 & 117 \end{bmatrix}$
$(A+B)^2 = A^2 + AB + BA + B^2$
$\because BA = \begin{bmatrix} 11 & 16 \\ 19 & 28 \end{bmatrix} \neq AB = \begin{bmatrix} 10 & 13 \\ 22 & 29 \end{bmatrix}\ \therefore (A+B)^2 - (A^2 + 2AB + B^2) = BA - AB \neq 0$
$\Rightarrow (A+B)^2 \neq A^2 + 2AB + B^2$

   (c) $A^2 + AB + AC + B^2 + BA + BC + C^2 + CA + CB = (A+B+C)^2 = 0^2 = 0$

5. $\textcircled{2}x + 3y + z = 8$ $x = 2$
$\textcircled{1}y + 3z = 4$ gives $y = 1$ If a zero is at the start of row 2 or 3
$\textcircled{8}z = 8$ $z = 1$ that avoids a row operation.

6. $EF = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$, $FE = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b+ac & c & 1 \end{bmatrix}$

$E^2 = \begin{bmatrix} 1 & 0 & 0 \\ 2a & 1 & 0 \\ 2b & 0 & 1 \end{bmatrix}$, $F^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3c & 1 \end{bmatrix}$

7. (a) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

8. Worked example 6 gives $|u_1||U_1| \leq \frac{1}{2}(u_1^2 + U_1^2)$ and $|u_2||U_2| \leq \frac{1}{2}(u_2^2 + U_2^2)$.
The whole line becomes $.96 \leq (.6)(.8)+(.8)(.6) \leq \frac{1}{2}\ (.6^2 + .8^2)+\frac{1}{2}(.8^2 + .6^2)$=1.
True: .96<1.

9. The combination $0w_1 + 0w_2 + 0w_3$ always gives the zero vector,but this problem looks for other zero combinations (then the vectors are dependent, they lie in a plane): $w_2 = (w_1 + w_3)/2$ so one combination that gives zero is $\frac{1}{2}w_1 - w_2 + \frac{1}{2}w_3$.