2018 Fall EECS205003 Linear Algebra - Homework 1

Name: ID:

1. Prove or explain mathematically that A is invertible if $a \neq 0$, and $a \neq b$ (find the pivots or A^{-1}):

$$A = \begin{bmatrix} a & b & b \\ a & a & b \\ a & a & a \end{bmatrix}.$$

2. Find and check the inverses (assuming they exist) of these block matrices:

$$\begin{bmatrix} I & 0 \\ C & I \end{bmatrix}, \begin{bmatrix} 0 & I \\ I & D \end{bmatrix}$$

3. Let $A = \begin{bmatrix} -1 & 2 & -2 \\ 4 & 3 & 0 \\ 7 & -3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 & 9 \\ -3 & 2 & 6 \\ -2 & -8 & 1 \end{bmatrix}$ be two 3×3 matrices. Please answer the following questions.

   (a) Compute AB using columns times rows

   (b) Let C be another 3×3 matrix and $\mathbf{c}_i$ denote the i-th columns of C.

   Suppose $A\mathbf{c}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $A\mathbf{c}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $A\mathbf{c}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. Try to compute AC.

4. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ be two 2×2 matrices. Please answer the following questions.

   (a) Compute $(A + B)^2$.

   (b) Compute $A^2 + 2AB + B^2$. Is the answer the same as (a) ? If it is not, explain the reason.

   (c) Let $C = \begin{bmatrix} -3 & -5 \\ -7 & -9 \end{bmatrix}$ be another 2×2 matrix.

   Compute $A^2 + AB + AC + B^2 + BA + BC + C^2 + CA + CB$.

5. Reduce this system to upper triangular form by two row operations:

$$\begin{aligned} 2x + 3y + z &= 8 \\ 4x + 7y + 5z &= 20 \\ -2y + 2z &= 0 \end{aligned}$$

   Circle the pivots. Solve by back substitution for z, y, x.

6. Multiply these matrices in the orders EF and FE:

$$\text{E}=\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{bmatrix} \quad \text{F}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 1 \end{bmatrix}$$

Also compute $E^2 = EE$ and $F^3 = FFF$. You can guess F^{100}.

7. (a) What 2 by 2 matrix R rotates every vector by 90°? R $\times \begin{bmatrix} x \\ y \end{bmatrix}$ is $\begin{bmatrix} y \\ -x \end{bmatrix}$

   (b) What 2 by 2 matrix R^2 rotates every vector by 180°?

8. Describe geometrically (line, plane, or all of R^3) all linear combination of
   (a) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 10 \\ 15 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$

9. Normally 4 "plane" in 4-dimensional space meet at a _____. Normally 4 column vectors in 4-dimensional space can combine to produces b. What combination of $(1,0,0,0), (1,1,0,0), (1,1,1,0)$, $(1,1,1,1)$ produces $b = (1,7,4,2)$? What 4 equations for x, y, z, t are you solving?

10. One-line proof of the Schwarz inequality $|u \cdot U| \leq 1$ for unit vectors:

$$|u \cdot U| \leq |u_1||U_1| + |u_2||U_2| \leq \frac{u_1^2+U_1^2}{2} + \frac{u_2^2+U_2^2}{2} = \frac{1+1}{2} = 1$$

   Put $(u_1, u_2) = (.6, .8)$ and $(U_1, U_2) = (.8, .6)$ in that whole line and find $\cos\theta$.

11. (a) Find the combination $x_1\mathbf{w}_1 + x_2\mathbf{w}_2 + x_3\mathbf{w}_3$ that gives the zero vector:

$$\mathbf{w}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \mathbf{w}_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \quad \mathbf{w}_3 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

   (b) Those vectors are (independent) (dependent).

   (c) The three vectors lie in a ______.

   (d) The matrix W with those columns is not invertible.