

# 2017 Fall EE203001 Linear Algebra - Homework 8

Due: None

1. (0%)
  - (a) What matrix transforms  $(1, 0)$  into  $(2, 5)$  and transforms  $(0, 1)$  to  $(1, 3)$ ?
  - (b) What matrix transforms  $(2, 5)$  to  $(1, 0)$  and  $(1, 3)$  to  $(0, 1)$ ?
  - (c) Why does no matrix transform  $(2, 6)$  to  $(1, 0)$  and  $(1, 3)$  to  $(0, 1)$ ?
  
2. (0%)
  - (a) What matrix  $M$  transforms  $(1, 0)$  and  $(0, 1)$  to  $(r, t)$  and  $(s, u)$ ?
  - (b) What matrix  $N$  transforms  $(a, c)$  and  $(b, d)$  to  $(1, 0)$  and  $(0, 1)$ ?
  - (c) What condition on  $a, b, c, d$  will make part (b) impossible?
  
3. (0%) Find the singular value decomposition of  $A = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$
  
4. (0%) Continue above question, Please find the pseudoinverse of  $A$  and the least square solution  $\hat{x}_0$  that  $b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  for  $A\hat{x}_0 = b$
  
5. (0%) The parabola  $\mathbf{w}_1 = \frac{1}{2}(x^2 + x)$  equals one at  $x = 1$ , and zero at  $x = 0$  and  $x = -1$ . Find the parabolas  $\mathbf{w}_2, \mathbf{w}_3$  from the conditions given below and then find  $y(x)$  using  $\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3$  by linearity.
  - (a)  $\mathbf{w}_2$  equals one at  $x = -1$ , and zero at  $x = 0$  and  $x = 1$ .
  - (b)  $\mathbf{w}_3$  equals one at  $x = 0$ , and zero at  $x = 1$  and  $x = -1$ .
  - (c)  $y(x)$  equals 9 at  $x = 1$  and 5 at  $x = 0$  and 7 at  $x = -1$ .
  
6. (0%) Suppose  $T$  is reflection across the  $45^\circ$  line, and  $S$  is reflection across the  $y$  axis. If  $\mathbf{v} = (a, b)$ , find  $S(T(\mathbf{v}))$  and  $T(S(\mathbf{v}))$ .
  
7. (0%) Suppose a linear  $T$  transforms  $(1,1)$  to  $(1,0,1)$  and  $(2,3)$  to  $(1,-1,4)$ . Find  $T(\mathbf{v})$ :
  - (a)  $v = (3, 4)$
  - (b)  $v = (4, 6)$
  - (c)  $v = (a, b)$
  
8. (0%) Given  $A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ ,  $B = \begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix}$ .  
If  $T(M) = AMB$ , please find  $T^{-1}(M)$  in the form  $( \quad )M( \quad )$ .