2017 Fall EE203001 Linear Algebra - Homework 7 solution

Due: 2018/1/5

1. (10%) Compute A^TA and AA^T and their eigenvalues and unit eigenvectors for V and U

$$\textbf{Rectangular} \quad \textbf{matrix} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

2. (10%) Suppose $(T(v_1) = w_1 + 2w_2 + 3w_3$ and$T(v_2) = 2w_2 + 3w_3$ and$T(v_3) = 3w_3$. Find the matrix A for T using these basis vectors. What input vector $vgivesT(v) = w_1$

3. (10%) Show that A and B are similar by finding M so that $B = M^{-1}AM$:

(a) $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 0 & -1 \\ 6 & 5 \end{bmatrix}$

(b) $A = \begin{bmatrix} 3 & -2 \\ 4 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 9 & 4 \\ -2 & 3 \end{bmatrix}$

4. (10%) Find the eigenvalues and unit eigenvectors v_1, v_2 of A^TA. Then find $u_1 = Av_1/\sigma$:

$A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ and $A^TA = \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix}$ and $AA^T = \begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}$

Verify that u_1 is a unit eigenvectors of AA^T. Complete the matrices U, Σ, V.

5. (10%)
   (a) If U and V are unitary matrices, show that U^{-1} and UV are also unitary.
   (b) A is a matrix with independent columns. Show that A^HA is not only Hermitian but also positive definite.

6. (10%) If A is a Hermitian matrix,show the property of its' real and imaginary part.(symmetric, Hermitian, ...etc.)

7. (10%) Which classes of matrices does P belong to: invertible, Hermitian, unitary? Compute P^2, P^3, and P^{100}. What are the eigenvalues of P?

$$P = \begin{bmatrix} 0 & i & 0 \\ 0 & 0 & i \\ i & 0 & 0 \end{bmatrix}.$$

8. (10%) Compute $\mathbf{y} = F_8\mathbf{c}$ by the three FFT steps for $\mathbf{c} = (1, 0, 1, 0, 1, 0, 1, 0)$. Repeat the computation for $c = (0, 1, 0, 1, 0, 1, 0, 1)$.

9. (10%) Prove that if $\boldsymbol{A}$ is a real symmetric matrix, then all eigenvalues of $\boldsymbol{A}$ are real numbers.

10. (10%) The columns of the Fourier matrix F are the *eigenvectors* of the cyclic permutation P. Multiply PF to find the eigenvalues λ_1 to λ_4:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^2 & {}^3 \\ 1 & i^2 & i^4 & {}^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^2 & {}^3 \\ 1 & i^2 & i^4 & {}^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix} \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \lambda_3 & \\ & & & \lambda_4 \end{bmatrix}$$

This is $PF = F\Lambda$ or $P = F\Lambda F^{-1}$.The eigenvector matrix(usually S) is F.