2017 Fall EE203001 Linear Algebra - Homework 6 solution

Due: 2017/12/22

1. (10%) For which s and t do A and B have all $\lambda > 0$ (therefore positive definite)?

$$A = \begin{bmatrix} s & -4 & -4 \\ -4 & s & -4 \\ -4 & -4 & s \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} t & 3 & 0 \\ 3 & t & 4 \\ 0 & 4 & t \end{bmatrix}$$

2. (10%) Find the eigenvalues and unit eigenvectors of A^TA and AA^T. Keep each $A\mathbf{v} = \sigma\mathbf{u}$:

$$\textbf{Fibonacci matrix } A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

Construct the singular value decomposition and verify that A equals $U\Sigma V^T$.

3. (10%) Write $\boldsymbol{A}$ in the form $\lambda_1 x_1 x_1^T + \lambda_2 x_2 x_2^T + \lambda_3 x_3 x_3^T$ of the spectral theorem $Q\Lambda Q^T$:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & -2 \\ 2 & -2 & 0 \end{bmatrix} \text{(keep } \|x_1\| = \|x_2\| = \|x_3\| = 1)$$

4. (10%) Without multiplying $\boldsymbol{A} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, find

   (a) (2%) the determinant of $\boldsymbol{A}$.

   (b) (2%) the eigenvalues of $\boldsymbol{A}$.

   (c) (2%) the eigenvectors of $\boldsymbol{A}$.

   (d) (4%) a reason why $\boldsymbol{A}$ is symmetric positive definite.

5. (10%) Find an orthogonal matrix Q that diagonalizes this symmetric matrix:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 4 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

6. (10%) Find the 3 by 3 matrix A and its pivots, rank, eigenvalues, and determinant:

$$A = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 4(x_1 - 2x_2 + x_3)^2$$

7. (10%)For which number b and c are these matrices positive definite?

$$A = \begin{bmatrix} 1 & b \\ b & 16 \end{bmatrix} \quad A = \begin{bmatrix} 2 & 3 \\ 4 & c \end{bmatrix}$$

With the pivots in D and multiplier in L, factor each A into $LDL^{\perp}$

8. (10%) What is the quadratic $f = ax^2 + 2bxy + cy^2$ for each of these matrices? Complete the square to write f as a sum of one or two squares $d_1(\quad)^2 + d_2(\quad)^2$.

$$A = \begin{bmatrix} 4 & 6 \\ 6 & 14 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 5 \\ 5 & 25 \end{bmatrix}$$

9. (10%) For a nearly symmetric matrix $A = \begin{bmatrix} 1 & 10^{-19} \\ 0 & 1 + 10^{-19} \end{bmatrix}$, find out how far are it's eigenvectors (in angle) from orthogonal.

10. (10%) Suppose A is a real antisymmetric matrix that $A^T = -A$, please show:

    (a) (3%) $\mathbf{x}^T A \mathbf{x} = 0$ for every real vector $\mathbf{x}$.

    (b) (4%) The eigenvalues of A are pure imaginary.

    (c) (3%) The determinant of A is non-negative.