2017 Fall EE203001 Linear Algebra - Homework 5

Due: 2017/12/8

1. (10%) Find two λ and $\boldsymbol{x}$ so that $\boldsymbol{y} = e^{\lambda t}\boldsymbol{x}$ solve

$$\frac{d\boldsymbol{y}}{dt} = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} y$$

What combination $\boldsymbol{y} = c_1 e^{\lambda_1 t}\boldsymbol{x}_1 + c_2 e^{\lambda_2 t}\boldsymbol{x}_2$ starts from $\boldsymbol{y}(0) = (0, 1) =?$

2. (10%) Find A to change the scalar equation $y'' = 4y' - 3y$ into a vector equation for $\boldsymbol{u} = (y, y')$:

$$\frac{d\boldsymbol{y}}{dt} = \begin{bmatrix} y' \\ y'' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} y \\ y' \end{bmatrix} = A\boldsymbol{u}$$

What are the eigenvalues of A? Find them also by substituting $y = e^{\lambda t}$ into $y'' = 4y' - 3y$.

3. (10%) Diagonalize the Matrix $B = \begin{bmatrix} n & 1 \\ 0 & n-1 \end{bmatrix}$. If $n = 2$, find b_{12} of B^{10}.

4. (10%) Suppose G_{k+2} is the average of the two previous numbers G_{k+1} and G_k:

$$\begin{aligned} G_{k+2} &= \tfrac{1}{2}G_{k+1} + \tfrac{1}{2}G_k \\ G_{k+1} &= G_k \end{aligned} \qquad \text{is} \qquad \begin{bmatrix} G_{k+2} \\ G_{k+1} \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} G_{k+1} \\ G_k \end{bmatrix}$$

   (a) Find the eigenvalues and eigenvectors of A.

   (b) Find the limit as $n \to \infty$ of the matrices $A^n = S\Lambda^n S^{-1}$.

5. (10%) Solve these linear equation by Cramer's Rule $\boldsymbol{x_j} = det(\boldsymbol{B_j})/\det(\boldsymbol{A})$

(a) $\begin{aligned} x_1 + 2x_2 &= 3 \\ 4x_1 + 5x_2 &= 6 \end{aligned}$ (b) $\begin{aligned} x_1 + 2x_2 &= 0 \\ 2x_1 + x_2 + 3x_3 &= 1 \\ 3x_2 + x_3 &= 0 \end{aligned}$

6. (10%) From the formula $\boldsymbol{AC^T} = (det\boldsymbol{A})\boldsymbol{I}$ show $(det\boldsymbol{C}) = (det\boldsymbol{A})^{n-1}$

7. (10%) The Block $\boldsymbol{B}$ has eigenvalue 3.5 and $\boldsymbol{C}$ has eigenvalue 1.2, and $\boldsymbol{D}$ has eigenvalues 3.9

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{B} & \boldsymbol{C} \\ \boldsymbol{O} & \boldsymbol{D} \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 & 1 \\ 1 & 5 & -2 & 3 \\ 0 & 0 & 6 & 3 \\ 0 & 0 & 3 & 6 \end{bmatrix}$$ Find the eigenvalue of $\boldsymbol{A} - \boldsymbol{2I}$.

8. (10%) $\boldsymbol{A} = \begin{bmatrix} 3 & -2 & 2 \\ 6 & -4 & 6 \\ 2 & -1 & 3 \end{bmatrix}$

   (a) Find the eigenvalues and eigenvector of $\boldsymbol{A}$.

   (b) Diagonalize the matrix $\boldsymbol{A}$.