2017 Fall EE203001 Linear Algebra - Homework 4

Due: 2017/11/24

1. (10%) For the closest parabola $b = C + Dt + Et^2$ to the four points $(0,0)$, $(1,8)$, $(3,8)$, and $(4,20)$. Write down the unsolvable equations $A\mathbf{x} = \mathbf{b}$ in three unknowns $\mathbf{x} = (C, D, E)$. Set up the three normal equations $A^T A\mathbf{x} = A^T\mathbf{b}$ (solution not required).

2. (10%) Use row operations to simplify and compute these determinants:

$$\det \begin{bmatrix} 101 & 201 & 301 \\ 102 & 202 & 302 \\ 103 & 203 & 303 \end{bmatrix} \text{ and } \det \begin{bmatrix} 1 & t & t^2 \\ t & 1 & t \\ t^2 & t & 1 \end{bmatrix}.$$

3. (10%) Given $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}$

   (a) Find orthonormal vectors $\mathbf{q}_1$, $\mathbf{q}_2$, $\mathbf{q}_3$ such that $\mathbf{q}_1$, $\mathbf{q}_2$ span the column space of $\mathbf{A}$.

   (b) Which of the four fundamental subspace contains $\mathbf{q}_3$?

   (c) Solve $\mathbf{Ax} = \mathbf{b}$ by least squares.

4. (10%) Find the determinant of A.

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & 3 & 5 \\ 4 & 16 & 9 & 25 \\ 8 & 64 & 27 & 125 \end{bmatrix}$$

5. (10%) Use the big formula with n! terms: $|A| = \sum \pm a_{1\alpha} a_{2\beta} \ldots . a_{n\omega}$ to compute the determinants of A, B, C from six terms. Are their rows independent?

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 5 & 1 & 3 \\ 5 & 3 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

6. (10%) Use cofactors $C_{ij} = (-1)^{i+j} det M_{ij}$. Find all cofactors of A and B and put them into cofactor matrices C, D. What is det(B)?

$$A = \begin{bmatrix} a & c \\ b & d \end{bmatrix}, B = \begin{bmatrix} 1 & 3 & 5 \\ 5 & 1 & 3 \\ 1 & 0 & 0 \end{bmatrix}$$

7. (10%) Consider the matrix

$$A = \begin{bmatrix} 3 & 3 & b \\ -2 & a & 7 \\ 9 & 5 & c \end{bmatrix} = LU \quad \text{where } U = \begin{bmatrix} 1 & d & e \\ 0 & -2 & f \\ 0 & 0 & 3 \end{bmatrix}$$

Please find the determinants of $L, U, A, U^{-1}L^{-1}$ and $U^{-1}L^{-1}A$.

8. (10%) Compute the determinants of these matrix by proper row/column operation:

$$A = \begin{bmatrix} 0 & a & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & c \\ d & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} a & a & a \\ a & b & b \\ a & b & c \end{bmatrix}$$

9. (10%) Find $\boldsymbol{q}_1$, $\boldsymbol{q}_2$, $\boldsymbol{q}_3$ (orthonormal) as combinations of $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ (independent columns of A). Then write A as QR:

$$A = \begin{bmatrix} 0 & 3 & 6 \\ 1 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$$

10. (10%) (a) Find a basis for the subspace $\mathbf{S}$ in $\mathbf{R}_4$ spanned by all solutions of $x_1 - x_2 - x_3 - x_4 = 0$ (b) Find a basis for the orthogonal complement $\mathbf{S}^{\perp}$. (c) Find $\boldsymbol{b}_1$ in $\boldsymbol{S}$ and $\mathbf{b}_2$ in $\boldsymbol{S}^{\perp}$ so that $\boldsymbol{b}_1 + \boldsymbol{b}_2 = h = (1, -1, -1, 1)$.