2017 Fall EE203001 Linear Algebra - Homework 3

Due: 2017/11/10

1. (10%) Suppose $\mathbf{S}$ is spanned by the vectors (1, 2, 1, 4) and (1, 3, 3, 4). Find two vectors that span $\mathbf{S}^{\perp}$. This is the same as solving $A\mathbf{x} = \mathbf{0}$ for which A ?

2. (10%) Project the vector $\mathbf{b}$ onto $\mathbf{a}$ to find $\mathbf{p}$. Let $\mathbf{e} = \mathbf{b} - \mathbf{p}$ and show that $\mathbf{e}$ is perpendicular to $\mathbf{a}$.

   (a) $\mathbf{b}$=(1, 2, 3), $\mathbf{a}$=(1, 0, 1).

   (b) $\mathbf{b}$=(1, 3, 5, 7), $\mathbf{a}$=(0, 1, 0, 1).

3. (12%) Project $\boldsymbol{b}$ = (0, 2, 8, 20) onto the line $\boldsymbol{a}$ = (2, 1, 1, 2). Find

   (a) $\hat{\boldsymbol{x}} = ?$

   (b) Projection $\boldsymbol{p} = ?$

   (c) Is $\boldsymbol{e} = \boldsymbol{b}$ - $\boldsymbol{p}$ perpendicular $\boldsymbol{a}$? Please explain it.

   (d) $||\boldsymbol{e}||$

4. (8%) Let $b = C + Dt$ be closest line to the points $(b, t) = (1, 2), (13, 4)$, and $(11, 3)$. Find the least squares solution $\hat{\boldsymbol{x}} = (C, D)$.

5. (10%) Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

   (a) Find a basis for the null space of A.

   (b) Given $\boldsymbol{x} = \begin{bmatrix} 2 & 3 & -1 \end{bmatrix}^T$, split it into a row space component $\boldsymbol{x_r}$ and $\boldsymbol{x_n}$.

6. (12%) If A is a matrix and $W^{\perp}$ is the orthogonal complement of a vector set W, which of the following are false? Why?

   (a) $W^{\perp}$ is always a subspace.

   (b) $C(A)^{\perp} = C(A^T)$

   (c) $C(A) = N(A)^{\perp}$

   (d) $C(A)^{\perp} = N(A^T)$

7. (8%) What linear combination of (-1, 1, 1) and (1, -1, 2) is closest to b = (3, -1, 7) ?

8. (10%) Suppose A is the 4 by 4 identity matrix with its last column removed. A is 4 by 3. Project $\mathbf{b} = (1, 3, 4, 2)$ onto the column space of A. What shape is the projection matrix P and what is P ?

9. (10%) Consider the matrix

$$P = \frac{1}{6}\begin{bmatrix} 5 & 2 & -1 \\ 2 & 2 & 2 \\ -1 & 2 & 5 \end{bmatrix}$$

Show that the length squared of column 2 equals P_{22}. Prove that the relation is true for any column n of P and P_{nn}. (Hint: use the properties of projection matrices).

10. (10%) Please prove the statement: If $A^T A\mathbf{x} = 0$, then $A\mathbf{x} = 0$ by examine which subspaces $A\mathbf{x}$ shall fall into.