EE205003 Session 7

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- Def The marix A is invertible if $\exists A^{-1}$ s.t. $A^{-1}A = I$ & $AA^{-1} = I$
- Notes
  Solving $A\mathbf{x} = \mathbf{b}$ is the same as finding A^{-1}!

  Note 1 :
  The inverse exists iff elimination produces n pivots (row exchanges allowed)
  ($A\mathbf{x} = \mathbf{b}$ is solvable)

  Note 2 :
  left inverse = right inverse
  $BA = I \nearrow$ $\nwarrow AC = I$
  $[B(AC) = (BA)C \Rightarrow BI = IC \Rightarrow B = C]$

Note 3 :

If A is invertible , $A\mathbf{x} = \mathbf{b}$ only has one sol : $\mathbf{x} = A^{-1}\mathbf{b}$

$(A^{-1}A\mathbf{x} = A^{-1}b \Rightarrow \mathbf{x} = A^{-1}\mathbf{b})$

Note 4 :

Suppose $\exists$ a nonzero vector x s.t. $A\mathbf{x} = \mathbf{0}$

$\Rightarrow A^{-1}$ does NOT exist

(not possible to have $A^{-1}(A\mathbf{x}) = \mathbf{x}$)

(If A invertible , $A\mathbf{x} = \mathbf{0}$ can only have zero sol. , i.e. , $\mathbf{x} = \mathbf{0}$)

Note 5 :

A 2x2 matrixis invertible iff $ad - bc \neq 0$

$\uparrow$ determinant

$$A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{|ad-bc|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Note 6 :

A diagonal matrix has an inverse if no diagonal entries are zero

$$A = \begin{bmatrix} d_1 & & \\ & \ddots & \\ & & d_n \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} \frac{1}{d_1} & & \\ & \ddots & \\ & & \frac{1}{d_n} \end{bmatrix}$$

Ex:

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

fail Note 1 (only have one pivot)

Note 4 ($A \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \mathbf{0}$)

Note 5 ($ad - bc = 0$)

Inverse of a product

Fact

A, B both invertible $\rightarrow AB$ invertible & $(AB)^{-1} = B^{-1}A^{-1}$

pf :$(B^{-1}A^{-1})(AB) = B^{-1}IB = I$)

(can be applied to 3 or more products

$(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$)

Ex: Inverse of elimination matrix

$$E = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(row 2 - 5row1) (row2 + 5row1)

(chk $EE^{-1} = I$)

For square matices,(left inverse is automatically a right inverse)

if $AB = I \Rightarrow BA = I$ $(B = A^{-1})$

Gauss-Jordan Elimination

For 3x3 matrix

$$AA^{-1} = A\begin{bmatrix}\mathbf{x}_1 & \mathbf{x}_2 & \mathbf{x}_3\end{bmatrix} = \begin{bmatrix}\mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3\end{bmatrix} = I$$

$\Rightarrow$solve 3 systems of eqns.

$$A\mathbf{x}_1 = \mathbf{e}_1 \quad A\mathbf{x}_2 = \mathbf{e}_2 \quad A\mathbf{x}_3 = \mathbf{e}_3$$

Augmented matrix:

$$\left[A|I\right] \rightarrow \left[I|E\right] \; (A \rightarrow U \rightarrow I)$$

$$(\Rightarrow E\left[A|I\right] = \left[I|E\right] \;\Rightarrow\; \left[EA|E\right] = \left[I|E\right] \;\Rightarrow\; E = A^{-1})$$

Ex:

$$K = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$\left[K|I\right] \;\rightarrow\; \left[I|K^{-1}\right]$$ (p. 84)