# EECS 205003 Session 4

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# Outline

**Ch2 Solving Linear Equations**

# The idea of Elimination

A systematic way to slove linear equations

Recall our Ex:

$$\begin{aligned} x - 2y &= 1 \Rightarrow x - 2y = 1 \text{ (equation } 1\times 3) \\ 3x + 2y &= 11 \Rightarrow \quad 8y = 8 \text{ (substract to climiinate 3x)} \end{aligned}$$

(upper triangular) $\uparrow$

($8y = 8 \Rightarrow y = 1$, plug in equation 1 $x = 2y + 1 = 2 \cdot 1 + 1 = 3$ )

(back substituton)

To eliminate x: substract a multiple of equation 1 from equation 2

(so that the system becomes triangular)

# The idea of Elimination

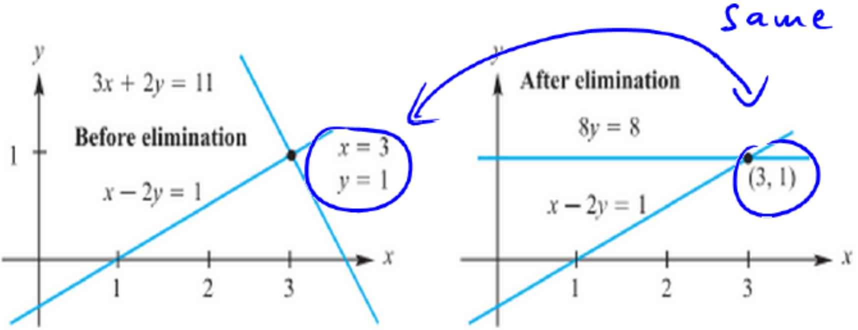


Figure 15: Eliminating $x$ makes the second line horizontal. Then $8y = 8$ gives $y = 1$.

# The idea of Elimination

$Q$: **How do you find the multiplier $l = 3$ ?**

(first pivot)

$$\begin{aligned} 1\,x - 2y &= 1 \Rightarrow x - 2y = 1 \\ 3\,x + 2y &= 11 \Rightarrow \quad 8y = 8 \end{aligned}$$

to eliminate 3x $\Rightarrow l = \frac{3}{1} = 3$

$$\begin{aligned} 4\,x - 8y &= 4 \Rightarrow 4x - 8y = 4 \\ 3\,x + 2y &= 11 \Rightarrow \quad 8y = 8 \end{aligned}$$

to eliminate 3x $\Rightarrow l = \frac{3}{4} = 3$

**Break down of Elimination**

Break down when zero in input

$\because$ no solution / too many solutions

Ex1:

$$\begin{array}{rl} x - 2y = 1 & \Rightarrow x - 2y = 1 \\ 3x - 6y = 11 & \Rightarrow \quad 0y = 8 \\ & \quad\ \Uparrow \end{array}$$

$2^{nd}$ pivot is zero $\Rightarrow$ fail !

(no solution $\because$ $0y = 0 \neq 8$)

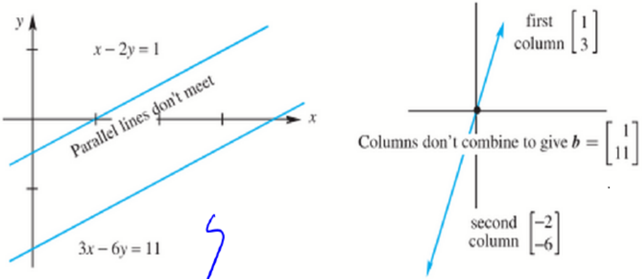


Figure 16: Row picture and column picture for Example 1: ***no solution***.

Row picture:

two lines are parallel

$\Rightarrow$ can never meet !

Column picture : $\begin{bmatrix} 1 \\ 3 \end{bmatrix} \begin{bmatrix} -2 \\ -6 \end{bmatrix}$ on the same line

all Combinations form a line but $\begin{bmatrix} 1 \\ 11 \end{bmatrix}$

in different direction

$\Rightarrow$ no combination can produce $\begin{bmatrix} 1 \\ 11 \end{bmatrix}$

Ex2: chang **b** = (1, 11) to (1, 3)

$$\begin{aligned} x - 2y = 1 &\Rightarrow x - 2y = 1 \\ 3x - 6y = 3 &\Rightarrow \quad 0y = 0 \end{aligned}$$

Zero in pivot ⇒ fail !   unknow y is free infinitely many solutions

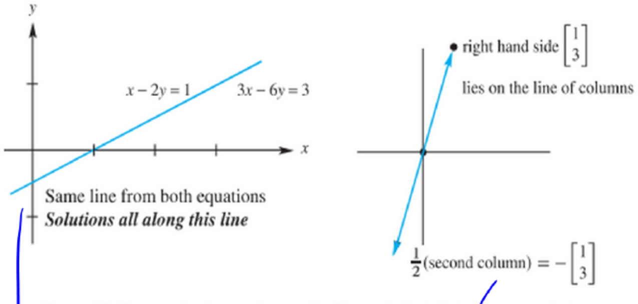


Figure 17: Row and column pictures for Example 2: *infinitely many solutions.*

Row picture:
a whole line of solutions

Column picture: **b** =(1, 3) same as column 1
so we can choose x = 1, y = 0

# The idea of Elimination

**In general**

For n equations, we don't get n pivots

$\Rightarrow$ Failure

$\Rightarrow$ Elimination leads to $\begin{cases} \mathbf{0} \neq \mathbf{0} & (no\ solution) \\ \mathbf{0} = 0 & (many\ solution) \end{cases}$

Success comes with n pivots but we may have to exchange the n equations.

# The idea of Elimination

Ex3: temporary failure but a row exchange fixes it

$\Downarrow \Leftarrow$ 0 in first pivot

$\Downarrow$

$$\begin{aligned} 0x + 2y &= 4 \Rightarrow 3x - 2y = 5 \\ 3x - 2y &= 5 \Rightarrow \qquad 2y = 4 \end{aligned}$$

(both row & column picture are normal but a row exchange is required)

# The idea of Elimination

**Three equations in three unknowns**

$$\begin{aligned} 2x + 4y - 2z &= 2 \\ 4x + 9y - 3z &= 8 \quad A\mathbf{x} = \mathbf{b} \\ -2x - 3y + 7z &= 10 \end{aligned}$$

$$\Downarrow$$

$$\begin{aligned} 2x + 4y - 2z &= 2 \\ 1y + 1z &= 4 \qquad \mathbf{U}\ \mathbf{x} = \mathbf{c} \\ 4z &= 8 \qquad \text{(upper triangular)} \end{aligned}$$

$\Downarrow \quad \Downarrow$

(hidden in the original system)

By back substitution ,

$4z = 8 \Rightarrow z = 2$ , $y + z = 4 \Rightarrow y = 2$ , $2x + 4y - 2z = 2 \Rightarrow x = -1$

**In general**

— use $1^{st}$ equation to create zeros below $1^{st}$ pivot
— use $2^{nd}$ equation to create zeros below $2^{nd}$ pivot
— keep going to find all n piovts and the triangular matrix **U**

multiplier $l_{ij} = (\frac{entry\ to\ eliminate\ in\ row\ i}{pivot\ in\ row\ j})$