

Homework #3
Coverage: Chapter 1–6
Due date: 5 June, 2019

Instructor: Chong-Yung Chi

TAs: Amin Jalili, Yi-Wei Li, Ping-Rui Chiang & Guei-Ming Liu

Notice:

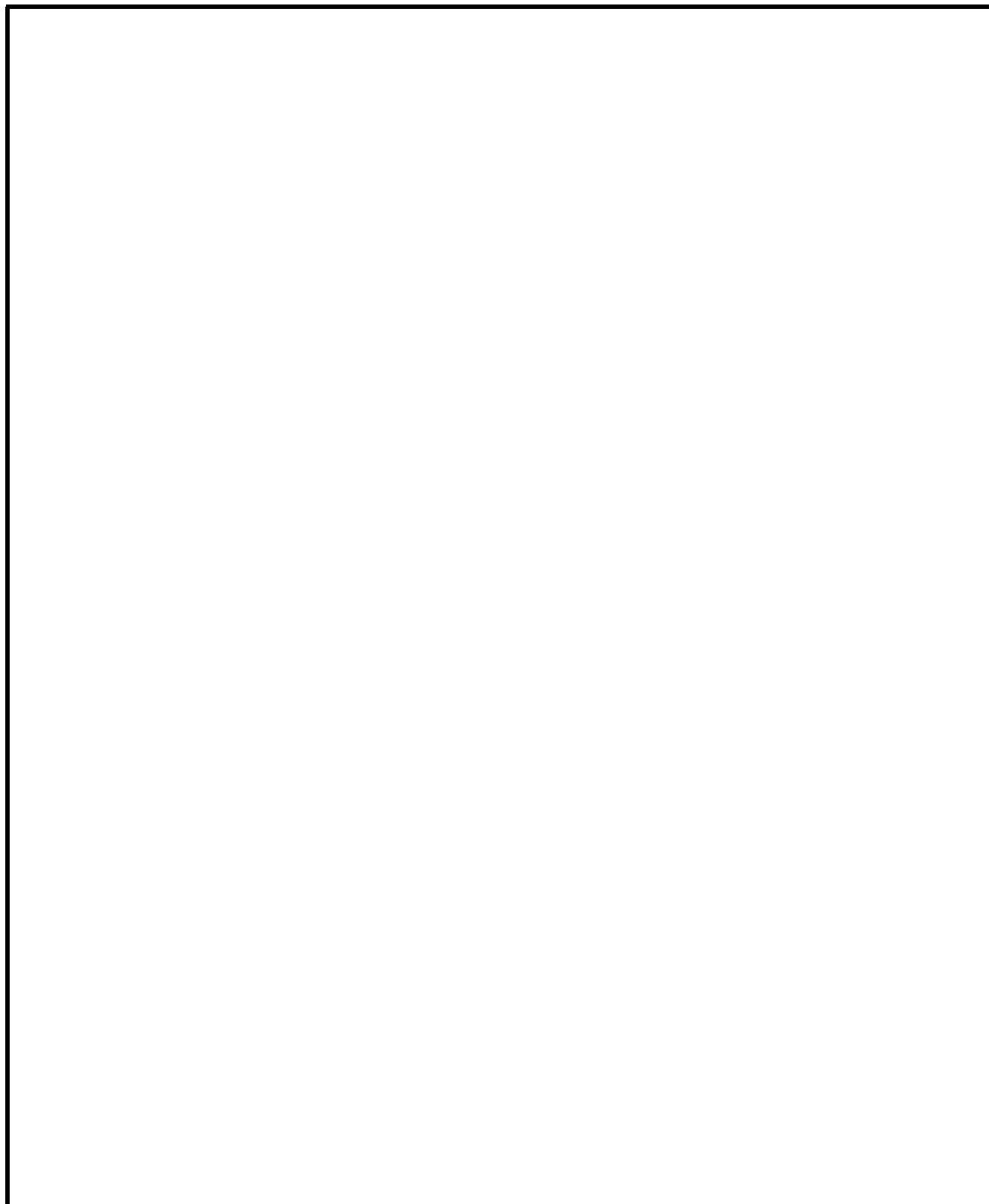
1. Please hand in your answer sheets by yourself to TAs in the class time or to the WCSP Lab., EECS building, R706, before 23:59 of the due date. **No late homework will be accepted.**
2. This homework includes **9 Problems** in **11 pages** with **100 points**.
3. Please justify your answers with clear, logical and solid reasoning or proofs.
4. You need to **print** the Problem Set and answer the problems in the **blank boxes** after each problem or sub-problem. We provided enough space for every problem. However, if you need more space, you can print it in one-side manner (each page in one side of an A4), and use the back side as an additional space.
5. Please do the homework independently by yourself. However, you may discuss with someone else but copied homework is not allowed. This will show your **respect toward the academic integrity**.
6. Write your name, student ID, email and department on the beginning of your answer sheets.
7. Your **legible handwriting** is fine. However, you are very welcome to use text formatting packages for writing your answers.

Name	
Student ID	
Department	
Email Address	

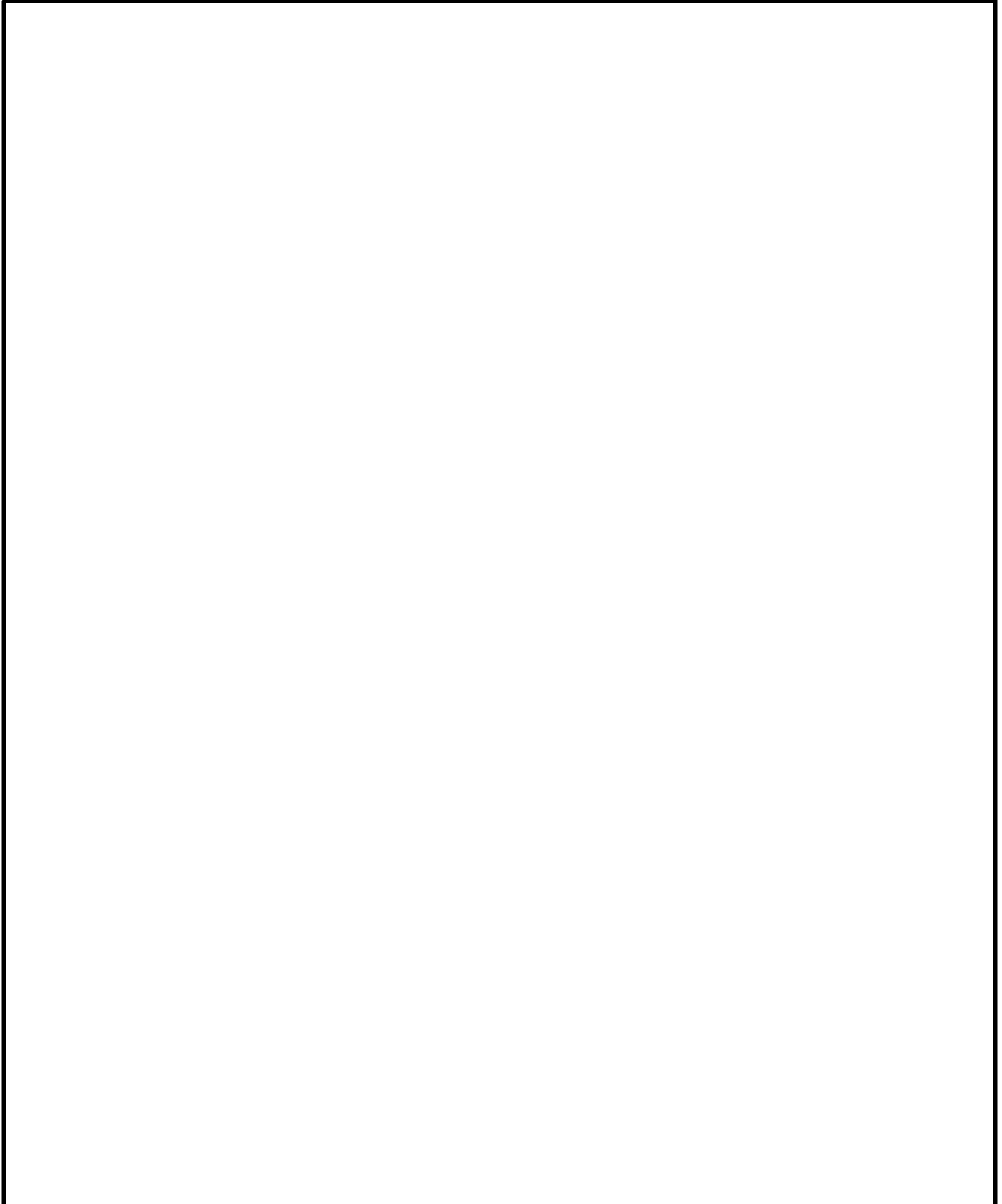
Problem	Score
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Problem 1. (20 points)

(a) (10 points) Let \mathbf{P} and \mathbf{Z} be positive semidefinite matrices such that $\mathbf{P}^2 = \mathbf{Z}^2$. Then $\mathbf{P} = \mathbf{Z}$.



- (b) (10 points) Let $\mathbf{A} \in \mathbb{R}^{n \times n}$ be invertible. Using the result in part (a), prove that there exist unique matrices \mathbf{Q} and \mathbf{P} which are orthogonal and positive-semidefinite matrices, respectively such that $\mathbf{A} = \mathbf{QP}$.



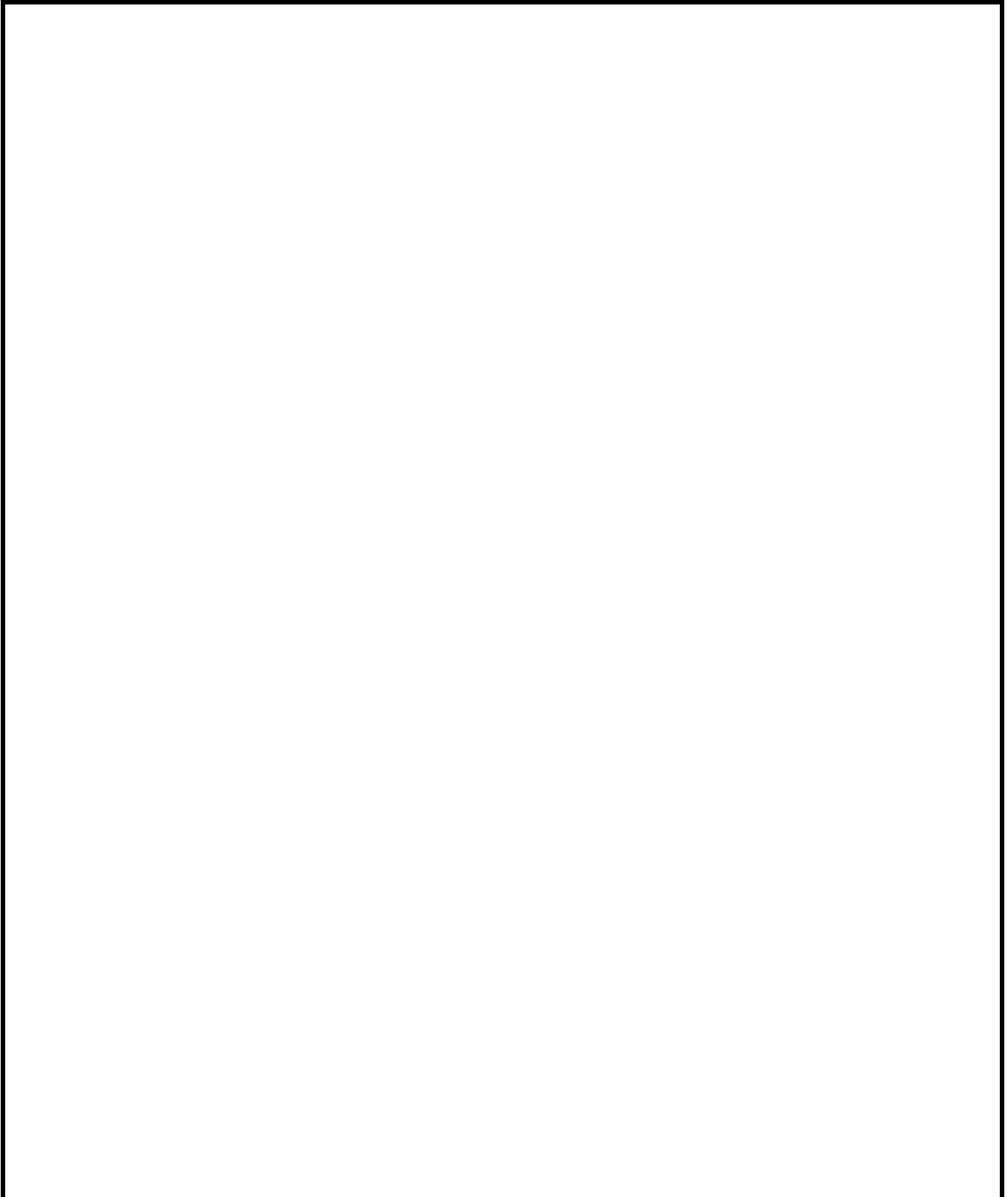
Problem 2. (15 points) Let \mathbf{A} be an invertible matrix.

(a) (5 points) Show that for any eigenvalue λ of \mathbf{A} , λ^{-1} is an eigenvalue of \mathbf{A}^{-1} .

(b) (5 points) Show that the eigenspace of \mathbf{A} corresponding to λ is the same as the eigenspace of \mathbf{A}^{-1} corresponding to λ^{-1} .

(c) (5 points) Show that if \mathbf{A} is invertible and diagonalizable, then \mathbf{A}^{-1} is also diagonalizable.

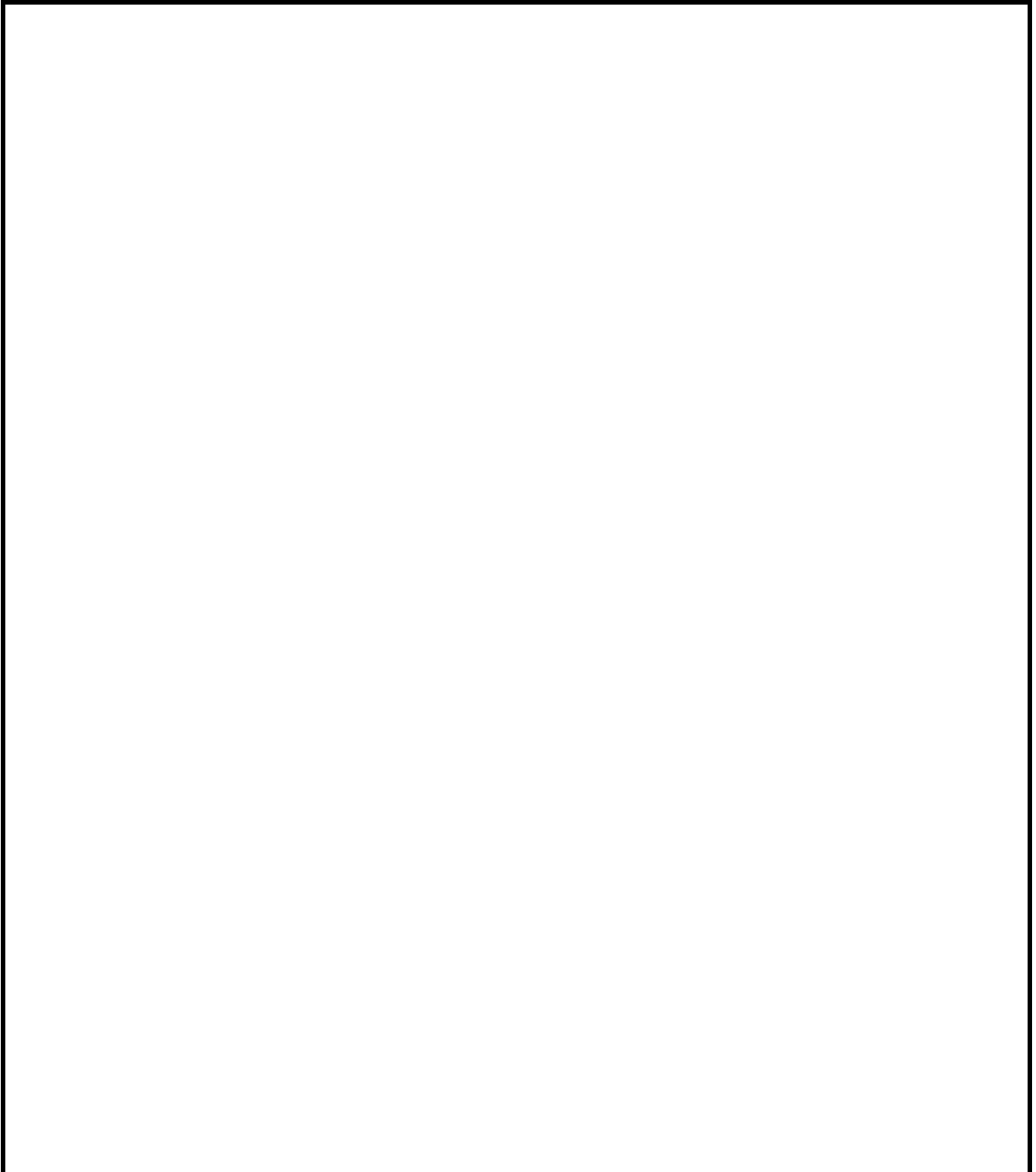
Problem 3. (15 points) The matrix $\mathbf{A} \in \mathbb{R}^{n \times n}$ can be written as $\mathbf{A} = \mathbf{L}\mathbf{D}\mathbf{U}$, where \mathbf{L} and \mathbf{U} are lower unitriangular and upper unitriangular matrices, respectively, and \mathbf{D} is a diagonal matrix. Show that LDU decomposition can be reduced to $\mathbf{A} = \mathbf{L}\mathbf{D}\mathbf{L}^T$ if \mathbf{A} is a nonsingular symmetric matrix.



Problem 4. (15 points) The Frobenius norm defined for $\mathbf{A} \in \mathbb{C}^{n \times n}$ by $\|\mathbf{A}\|_F = (\text{Tr}(\mathbf{A}^H \mathbf{A}))^{1/2}$ where $\text{Tr}(\cdot)$ denotes the trace of a matrix. Show that

$$\|\mathbf{A}\|_F \leq (\text{rank}(\mathbf{A}))^{1/2} \|\mathbf{A}\|_2,$$

where $\|\mathbf{A}\|_2 = \max_{\|\mathbf{x}\|_2=1} \|\mathbf{A}\mathbf{x}\|_2$ and \mathbf{x} is an $n \times 1$ vector.

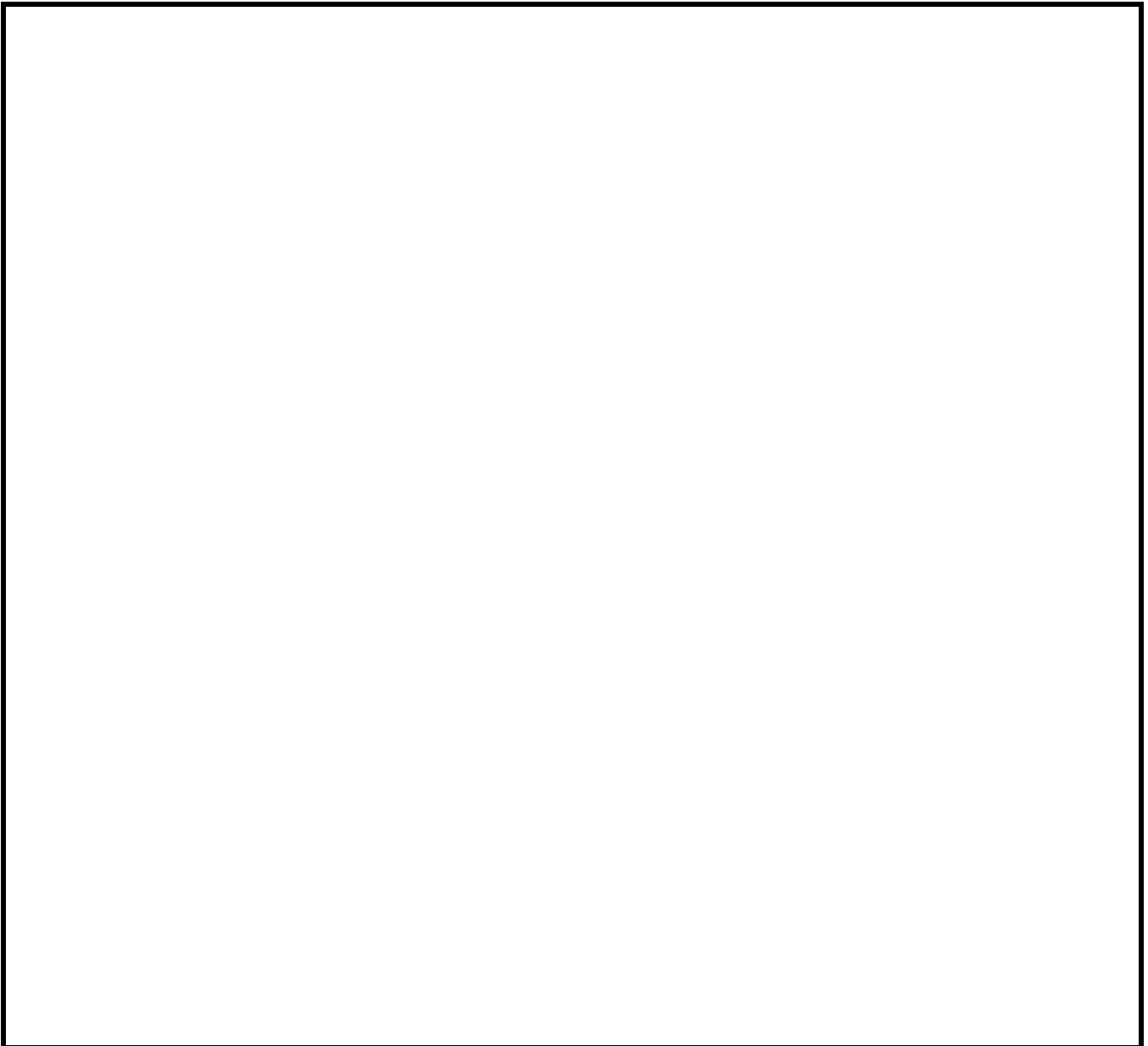


Problem 5. (5 points) Let $\mathbf{A} = \begin{bmatrix} 6 & -1 \\ 3 & 2 \end{bmatrix}$. Find a general formula based on n (a positive integer) for \mathbf{A}^n .

Problem 6. (5 points) Define the matrix \mathbf{A} as

$$n \times 2n \left\{ \begin{array}{l} \left[\begin{array}{cccccccc} 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 0 \\ 9 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ 0 & 9 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 9 & 0 & 0 & \cdots & 1 \end{array} \right] \\ \underbrace{\hspace{10em}}_{2n \times n} \end{array} \right.$$

be a $2n \times 2n$ matrix. Find \mathbf{A}^ℓ where ℓ is a positive integer number.



Problem 7. (5 points) Let V be a finite dimensional vector space with $\dim(V) = n$. Prove that every basis for V contains the same number of vectors.

Problem 8. (10 points) Let $\mathbf{A} = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 3 & 1 \\ 2 & -1 & 4 \end{bmatrix}$. Find \mathbf{J} and invertible \mathbf{S} such that $\mathbf{J} = \mathbf{S}^{-1}\mathbf{A}\mathbf{S}$ where \mathbf{J} is the Jordan canonical form of \mathbf{A} .

Problem 9. (10 points) Consider the matrix $\mathbf{A} = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$.

- (a) Find the complete solution for the equation $\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x}(t)$ with the initial condition $\mathbf{x}(0) = [0 \ 2]^T$.
- (b) Find the singular value decomposition as $\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$ using diagonalization of the matrix $\mathbf{A}^T\mathbf{A}$.

