EECS205000: Linear Algebra **Spring 2019**
College of Electrical Engineering and Computer Science
National Tsing Hua University

Homework #3
Coverage: Chapter 1–6
Due date: 5 June, 2019

Instructor: Chong-Yung Chi *TAs: Amin Jalili, Yi-Wei Li, Ping-Rui Chiang & Guei-Ming Liu*

Notice:

1. *Please hand in your answer sheets by yourself to TAs in the class time or to the WCSP Lab., EECS building, R706, before 23:59 of the due date.* ***No late homework will be accepted.***
2. *This homework includes* ***9 Problems*** *in* ***11 pages*** *with* ***100 points****.*
3. *Please justify your answers with clear, logical and solid reasoning or proofs.*
4. *You need to* ***print*** *the Problem Set and aanswer the problems in the* ***blank boxes*** *after each problem or sub-problm. We provided enough space for every problem. However, if you need more space, you can print it in one-side manner (each page in one side of an A4), and use the back side as an aditional space.*
5. *Please do the homework independently by yourself. However, you may discuss with someone else but copyied homework is not allowed. This will show your* ***respect toward the academic integrity****.*
6. *Write your name, student ID, email and department on the begining of your ansewr sheets.*
7. *Your* ***legible handwriting*** *is fine. However, you are very welcome to use text formatting packages for writing your answers.*

Name	
Student ID	
Department	
Email Address	

Problem	Score
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Problem 1. (20 points)

(a) (10 points) Let $\mathbf{P}$ and $\mathbf{Z}$ be positive semidefinite matrices such that $\mathbf{P}^2 = \mathbf{Z}^2$. Then $\mathbf{P} = \mathbf{Z}$.

(b) (10 points) Let $\mathbf{A} \in \mathbb{R}^{n\times n}$ be invertible. Using the result in part (a), prove that there exist unique matrices $\mathbf{Q}$ and $\mathbf{P}$ which are orthogonal and positive-semidefinite matrices, respectively such that $\mathbf{A} = \mathbf{QP}$.

Problem 2. (15 points) Let $\mathbf{A}$ be an invertible matrix.

(a) (5 points) Show that for any eigenvalue λ of $\mathbf{A}$, λ^{-1} is an eigenvalue of $\mathbf{A}^{-1}$.

(b) (5 points) Show that the eigenspace of $\mathbf{A}$ corresponding to λ is the same as the eigenspace of $\mathbf{A}^{-1}$ corresponding to λ^{-1}.

(c) (5 points) Show that if $\mathbf{A}$ is invertible and diagonalizable, then $\mathbf{A}^{-1}$ is also diagonalizable.

Problem 3. (15 points) The matrix $\mathbf{A} \in \mathbb{R}^{n \times n}$ can be written as $\mathbf{A} = \mathbf{LDU}$, where $\mathbf{L}$ and $\mathbf{U}$ are lower unitriangular and upper unitriangular matrices, respectively, and $\mathbf{D}$ is a diagonal matrix. Show that LDU decomposition can be reduced to $\mathbf{A} = \mathbf{LDL}^T$ if $\mathbf{A}$ is a nonsingular symmetric matrix.

Problem 4. (15 points) The Frobenius norm defined for $\mathbf{A} \in \mathbb{C}^{n\times n}$ by $\|\mathbf{A}\|_F = \left(\text{Tr}(\mathbf{A}^H\mathbf{A})\right)^{1/2}$ where $\text{Tr}(\cdot)$ denotes the trace of a matrix. Show that

$$\|\mathbf{A}\|_F \leq (\text{rank}(\mathbf{A}))^{1/2}\|\mathbf{A}\|_2,$$

where $\|\mathbf{A}\|_2 = \max_{\|\mathbf{x}\|_2=1} \|\mathbf{A}\mathbf{x}\|_2$ and $\mathbf{x}$ is an $n \times 1$ vector.

Problem 5. (5 points) Let $\mathbf{A} = \begin{bmatrix} 6 & -1 \\ 3 & 2 \end{bmatrix}$. Find a general formula based on n (a positive integer) for $\mathbf{A}^n$.

Problem 6. (5 points) Define the matrix $\mathbf{A}$ as

$$\begin{matrix} n \times 2n \left\{ \vphantom{\begin{matrix} 1 \\ 0 \\ \vdots \\ 0 \end{matrix}} \right. \\ \\ \\ \\ \\ \end{matrix} \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 0 \\ 9 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ 0 & 9 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 9 & 0 & 0 & \cdots & 1 \end{bmatrix} \\ \underbrace{\hphantom{0 \;\; 0 \;\; \cdots \;\; 9}}_{2n \times n}$$

be a $2n \times 2n$ matrix. Find $\mathbf{A}^\ell$ where ℓ is a positive integer number.

Problem 7. (5 points) Let V be a finite dimensional vector space with $dim(V) = n$. Prove that every basis for V contains the same number of vectors.

Problem 8. (10 points) Let $\mathbf{A} = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 3 & 1 \\ 2 & -1 & 4 \end{bmatrix}$. Find $\mathbf{J}$ and invertible $\mathbf{S}$ such that $\mathbf{J} = \mathbf{S}^{-1}\mathbf{A}\mathbf{S}$ where $\mathbf{J}$ is the Jordan canonical form of $\mathbf{A}$.

Problem 9. (10 points) Consider the matrix $\mathbf{A} = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$.

(a) Find the complete solution for the equation $\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x}(t)$ with the initial condition $\mathbf{x}(0) = [0 \ \ 2]^T$.

(b) Find the singular value decomposition as $\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$ using diagonalization of the matrix $\mathbf{A}^T\mathbf{A}$.