2017 Fall EE203001 Linear Algebra - Quiz 9 (solution)

Name: ID:

There are three recursive equation $\begin{cases} x_n = x_{n-1} + 0.5y_{n-1} \\ y_n = 0.5y_{n-1} + z_{n-1} \\ z_n = 0 \end{cases}$ can be represented by $\begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix} = A \begin{bmatrix} x_{n-1} \\ y_{n-1} \\ z_{n-1} \end{bmatrix}$

(a) Find the eigenvalues and corresponding eigenvectors of A

(b) Diagonalize the Matrix A

(c) Find A^{∞}

sol :

(a) $\det(A - \lambda I) = -\lambda(0.5 - \lambda)(1 - \lambda) = 0 \to \lambda = 0, 0.5, 1$

$$\lambda = 0 \to \bar{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$\lambda = 0.5 \to \bar{v}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\lambda = 1 \to \bar{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

(b) $P = [\bar{v}_1, \bar{v}_2, \bar{v}_3] = \begin{bmatrix} 1 & -1 & 1 \\ -2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$$AP = [A\bar{v}_1, A\bar{v}_2, A\bar{v}_3] = [0, 0.5\bar{v}_2, \bar{v}_3] = [\bar{v}_1, \bar{v}_2, \bar{v}_3] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} = PD$$

$$P^{-1}AP = D$$

(c) $A^{\infty} = PD^{\infty}P^{-1} = P \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} P^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$