2017 Fall EE203001 Linear Algebra - Quiz 7 (solution)

Name: ID:

1. $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 1 & 4 & 6 \end{bmatrix}$, find the QR decomposition of A.

Ans:

(1) $A = \begin{bmatrix} \boldsymbol{c}_1 & \boldsymbol{c}_2 & \boldsymbol{c}_3 \end{bmatrix} \rightarrow$ Find Q has orthonormal column.

step 1.

(i) $\boldsymbol{\phi}_1 = \boldsymbol{c}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \| \boldsymbol{\phi}_1 \|^2 = 2$

(ii) $\boldsymbol{\phi}_2 = \boldsymbol{c}_2 - \dfrac{\boldsymbol{c}_2^T \boldsymbol{\phi}_1}{\| \boldsymbol{\phi}_1 \|^2} \boldsymbol{\phi}_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \| \boldsymbol{\phi}_2 \|^2 = 3$

(iii) $\boldsymbol{\phi}_3 = \boldsymbol{c}_3 - \dfrac{\boldsymbol{c}_3^T \boldsymbol{\phi}_2}{\| \boldsymbol{\phi}_2 \|^2} \boldsymbol{\phi}_2 - \dfrac{\boldsymbol{c}_3^T \boldsymbol{\phi}_1}{\| \boldsymbol{\phi}_1 \|^2} \boldsymbol{\phi}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

step 2.

(i) $\boldsymbol{q}_1 = \dfrac{\boldsymbol{\phi}_1}{\| \boldsymbol{\phi}_1 \|} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

(ii) $\boldsymbol{q}_2 = \dfrac{\boldsymbol{\phi}_2}{\| \boldsymbol{\phi}_2 \|} = \frac{1}{\sqrt{3}} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

$$\Rightarrow Q = \begin{bmatrix} \dfrac{1}{\sqrt{2}} & -\dfrac{1}{\sqrt{3}} \\ 0 & \dfrac{1}{\sqrt{3}} \\ \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{3}} \end{bmatrix}$$

(2) $A = QR$

$$\Rightarrow R = Q^T A = \begin{bmatrix} \sqrt{2} & 3\sqrt{2} & 5\sqrt{2} \\ 0 & \sqrt{3} & \sqrt{3} \end{bmatrix}$$