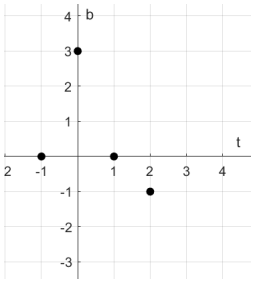
2017 Fall EE203001 Linear Algebra - Quiz 6

Name: ID:

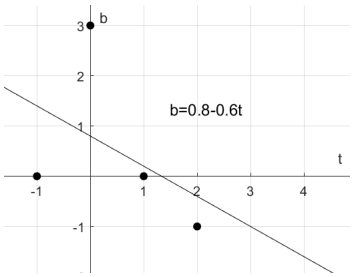
1. Given four points $(-1,\ 0)$, $(2,\ -1)$, $(0,\ 3)$, and $(1,\ 0)$.



   (a) Please find the closest line $b = C + Dt$ through these points.

   (b) What is the total vertical distance to this line?

Sol:



(a)

$$\begin{array}{llll} C & -D & = 0 \\ C & +2D & = -1 \\ C & & = 3 \\ C & +D & = 0 \end{array}$$

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 2 \\ 1 & 0 \\ 1 & 1 \end{bmatrix},\ \hat{\mathbf{x}} = \begin{bmatrix} C \\ D \end{bmatrix},\ \mathbf{b} = \begin{bmatrix} 0 \\ -1 \\ 3 \\ 0 \end{bmatrix}$$

Solve normal equation $A^T A\hat{\mathbf{x}} = A^T\mathbf{b}$.

$$\begin{bmatrix} 4 & 2 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix},$$

Get $C = 0.8$, $D = -0.6$, thus $b = 0.8 - 0.6t$ is the closest line.

(b)
The distances to $b = 0.8 - 0.6t$ for each point is:

$$\begin{aligned}\mathbf{e} =& \mathbf{b} - \mathbf{p} \\ =& \mathbf{b} - A\hat{\mathbf{x}} \\ =& \begin{bmatrix} 0 \\ -1 \\ 3 \\ 0 \end{bmatrix} - \begin{bmatrix} 1.4 \\ -0.4 \\ 0.8 \\ 0.2 \end{bmatrix} \\ =& \begin{bmatrix} -1.4 \\ -0.6 \\ 2.2 \\ 0.2 \end{bmatrix}\end{aligned}$$

The total vertical distance is the sum of absolute values of all elements of $\mathbf{e}$, which is 4.4.