

2017 Fall EE203001 Linear Algebra - Quiz 5

Name:

ID:

1. Given S is a subspace of R^4 .

(a) If S contains only the zero vector, what is S^\perp ?

(b) If S is spanned by $(1, -1, 0, 1)$ and $(0, 1, -1, -1)$, what is the basis for S^\perp ?

Sol:

(a) S^\perp is R^4 .

$$(b) \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix} \mathbf{x} = \mathbf{0} \Rightarrow \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & -1 \end{bmatrix} \mathbf{x} = \mathbf{0}$$

Let $x_3 = c_1$ and $x_4 = c_2$. Then $x_1 = c_1$ and $x_2 = c_1 + c_2$.

$$S^\perp = c_1(1, 1, 1, 0) + c_2(0, 1, 0, 1)$$

S^\perp is spanned by $(1, 1, 1, 0)$ and $(0, 1, 0, 1)$.

Note that S^\perp is also spanned by $(1, 0, 1, -1)$ and $(0, 1, 0, 1)$. (Let $x_1 = c_1$ and $x_2 = c_2$.)