2017 Fall EE203001 Linear Algebra - Homework 2

Due: 2017/10/20

1. (8%) Use Gauss-Jordan elimination on $\begin{bmatrix} U & I \end{bmatrix}$ to find the upper triangular U^{-1}:

$$UU^{-1} = I \qquad \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \vec{\boldsymbol{x}}_{\boldsymbol{1}} & \vec{\boldsymbol{x}}_{\boldsymbol{2}} & \vec{\boldsymbol{x}}_{\boldsymbol{3}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

2. (12%) Suppose A is already lower triangular with 1 's on the diagonal. $A = L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}.$

   The elimination matrices E_{21} , E_{31} , E_{32} contain $-a$ then $-b$ then $-c$.
   (a) Multiply E_{21} E_{31} E_{32} to find the single matrix E that produces $EA = I$.
   (b) Multiply E_{21}^{-1} E_{31}^{-1} E_{32}^{-1} to bring back L (nicer than E).

3. (10%) For which three numbers c is this matrix not invertible, and why not?

$$A = \begin{bmatrix} 5 & c & c \\ 3 & 4 & c \\ c & c & c \end{bmatrix}$$

4. (10%) A and B are symmetric across the diagonal (because 6=6,2=2). Find their triple factorizations LDU and say how U is related to L for these symmetric matrices:

$$A = \begin{bmatrix} 2 & 6 \\ 6 & 20 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 5 & 2 \\ 0 & 2 & 9 \end{bmatrix}$$

5. (8%) Find A^{-1} and B^{-1}(if they exist) by elimination on $[A \quad I]$ and $[B \quad I]$:

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

6. (10%) We can actually solve $A\boldsymbol{x} = \boldsymbol{b}$ without needing explicit A. First, you are asked to solve $L\boldsymbol{c} = \boldsymbol{b}$ to find $\boldsymbol{c}$. Then solve $U\boldsymbol{x} = \boldsymbol{c}$ to find $\boldsymbol{x}$. Briefly explain the concept behind these steps.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \boldsymbol{b} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

7. (12%) Given $Z = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$. Compute (a)$Z - Z^T$ (b)Z^3 (c)Z^{2n} n=1,2,3,....

8. (8%) Let $Q = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$. Find Q^{-1}

9. (12%) Suppose you eliminate upwards (almost unheard of.) Use the last row to produce zeros in the column (the pivot is 1). Then use the second row to produce zero above the second pivot. Find the factors in the unusual order A =UL

$$\textbf{Upper times lower} \qquad \mathbf{A} = \begin{bmatrix} 6 & 3 & 2 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

10. (10%) Which permutation makes PA upper triangular? Which permutations make $P_1 A P_2$ lower triangular? Multiplying A on the right by P_2 exchanges the ________ of A.

$$A = \begin{bmatrix} 0 & 0 & 9 \\ 4 & 8 & 7 \\ 0 & 4 & 1 \end{bmatrix}$$