2017 Fall EE203001 Linear Algebra - Homework 1

Due: 2017/10/6

1. (12%) Calculate the dot product $\vec{u} \cdot \vec{v}$ and $\vec{u} \cdot \vec{w}$ and $\vec{u} \cdot (\vec{v} + \vec{w})$ and $\vec{w} \cdot \vec{v}$.

$$\vec{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} 5 \\ 6 \end{bmatrix} \qquad \vec{w} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

2. (14%) Normally 4 "plane" in 4-dimensional space meet at a _____. Normally 4 column vectors in 4-dimensional space can combine to produces b. What combination of $(1, 0, 0, 0), (1, 1, 0, 0), (1, 1, 1, 0), (1, 1, 1, 1)$ produces $b = (1, 7, 4, 2)$? What 4 equations for x, y, z, t are you solving?

3. (12%) Draw the row and column pictures for the equations $x + y = 0$, $x - y = 2$.

4. (12%) Choose a coefficient b that makes this system singular. Then choose a right side g that makes it solvable. Find two solutions in that singular case.

$$\begin{aligned} x + by &= 4 \\ 2x + 2y &= g. \end{aligned}$$

5. (14%) In the xy plane, draw the lines $x + y = 5$ and $x + 2y = 6$ and the equation $y =$ _____ that comes from elimination. The line $5x - 4y = c$ will go through the solution of these equation if $c =$ _____.

6. (12%) Which number q makes this system singular and which right side t gives if infinitely many solutions? Find the solution that has $z = 1$.

$$\begin{aligned} x + 4y - 2z &= 1 \\ x + 7y - 6z &= 6 \\ 3y + qz &= t \end{aligned}$$

7. (12%) For which three numbers a will elimination fail to give three pivots?

$$A = \begin{bmatrix} a & 2 & 3 \\ a & a & 4 \\ a & a & a \end{bmatrix} \text{ is singular for three values of a.}$$

8. (12%) Apply elimination to the 3 by 4 augmented matrix $[Ab]$. How do you know this system has no solution? Change the last number 6 so there is a solution.

$$Ax = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}.$$