5.1

31. (a) A four-engine plane is preferable to a two-engine plane if and only if

$$1-\binom{4}{0}p^0(1-p)^4-\binom{4}{1}p(1-p)^3>1-\binom{2}{0}p^0(1-p)^2.$$

This inequality gives $p>2/3$. Hence a four-engine plane is preferable if and only if $p>2/3$. If $p=2/3$, it makes no difference.
(b) A five-engine plane is preferable to a three-engine plane if and only if

$$\binom{5}{5}p^5(1-p)^0+\binom{5}{4}p^4(1-p)+\binom{5}{3}p^3(1-p)^2>\binom{3}{2}p^2(1-p)+p^3.$$

Simplifying this inequality, we get $3(p-1)^2(2p-1)\geq 0$ which implies that a five-engine plane is preferable if and only if $2p-1\geq 0$. That is, for $p>1/2$, a five-engine plane is preferable; for $p<1/2$, a three-engine plane is preferable; for $p=1/2$ it makes no difference.

5.2

23. The expected number of fractures per meter is $\lambda=1/60$. Let $N(t)$ be the number of fractures in t meters of wire. Then

$$P\big(N(t)=n\big)=\frac{e^{-t/60}(t/60)^n}{n!},\quad n=0,1,2,\ldots.$$

In a ten minute period, the machine turns out 70 meters of wire. The desired probability, $P\big(N(70)>1\big)$ is calculated as follows:

$$\begin{aligned}P\big(N(70)>1\big)&=1-P\big(N(70)=0\big)-P\big(N(70)=1\big)\\&=1-e^{-70/60}-\frac{70}{60}e^{-70/60}\approx 0.325.\end{aligned}$$

33. (a) Let A be the region whose points have a (positive) distance d or less from the given tree. The desired probability is the probability of no trees in this region and is equal to

$$\frac{e^{-\lambda\pi d^2}(\lambda\pi d^2)^0}{0!}=e^{-\lambda\pi d^2}.$$

(b) We want to find the probability that the region A has at most $n-1$ trees. The desired quantity is

$$\sum_{i=0}^{n-1}\frac{e^{-\lambda\pi d^2}(\lambda\pi d^2)^i}{i!}.$$

5.3

21. The probability that the sixth coin is accepted on the nth try is

$$\binom{n-1}{5}(0.10)^6(0.90)^{n-6}.$$

Therefore, the desired probability is

$$\sum_{n=50}^{\infty}\binom{n-1}{5}(0.10)^6(0.90)^{n-6}=1-\sum_{n=6}^{49}\binom{n-1}{5}(0.10)^6(0.90)^{n-6}=0.6346.$$

23. If the fifth tails occurs after the 14th trial, ten or more heads have occurred. Therefore, the fifth tail occurs before the tenth head if and only if the fifth tail occurs before or on the 14th flip. Calling tails success, X, the number of flips required to get the fifth tail is negative binomial with parameters 5 and $1/2$. The desired probability is given by

$$\sum_{n=5}^{14}P(X=n)=\sum_{n=5}^{14}\binom{n-1}{4}\Big(\frac{1}{2}\Big)^5\Big(\frac{1}{2}\Big)^{n-5}\approx 0.91.$$