國立清華大學
NATIONAL TSING HUA UNIVERSITY

EE 306001
Probability

Lecture 6-2: discrete random variable

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Lecture outline

Readings: Section 2.1 – 2.3

- Random variables
- Probability mass function (PMF)

We have covered all the basics foundations of probability

- Now, we are going to introduce a concept of random variable
  - Ways to assign numerical results to the outcomes of an experiment
- We will define what are random variables, describe random variables using probability mass function (PMF)
  - PMF: a compact way of representing how some numerical values are more likely to occur compared to others
  - Think about going from sets -> algebra

Random variables

- An assignment of a value (number) to every possible outcome
  - Imagine the sample space is this class (many students in this class)
  - We are interested in the height of a random student
  - Measuring the height using ‘real number’
  - Then, carry out the experiment

- Mathematically:
  - A function from the sample space Ω to the real numbers
  - Imaging this function takes an argument, which is the outcome of the experiment, i.e., a typical student, and produces the value of that function, which is the height of that particular student
- Notation-wise (note the distinction)
  - Random variable, ***X*** – this is a function
  - Numerical value, ***x*** – this is a value that the function takes
- Can have multiple random variables for the same sample space
  - In this example, say, the weight of a student

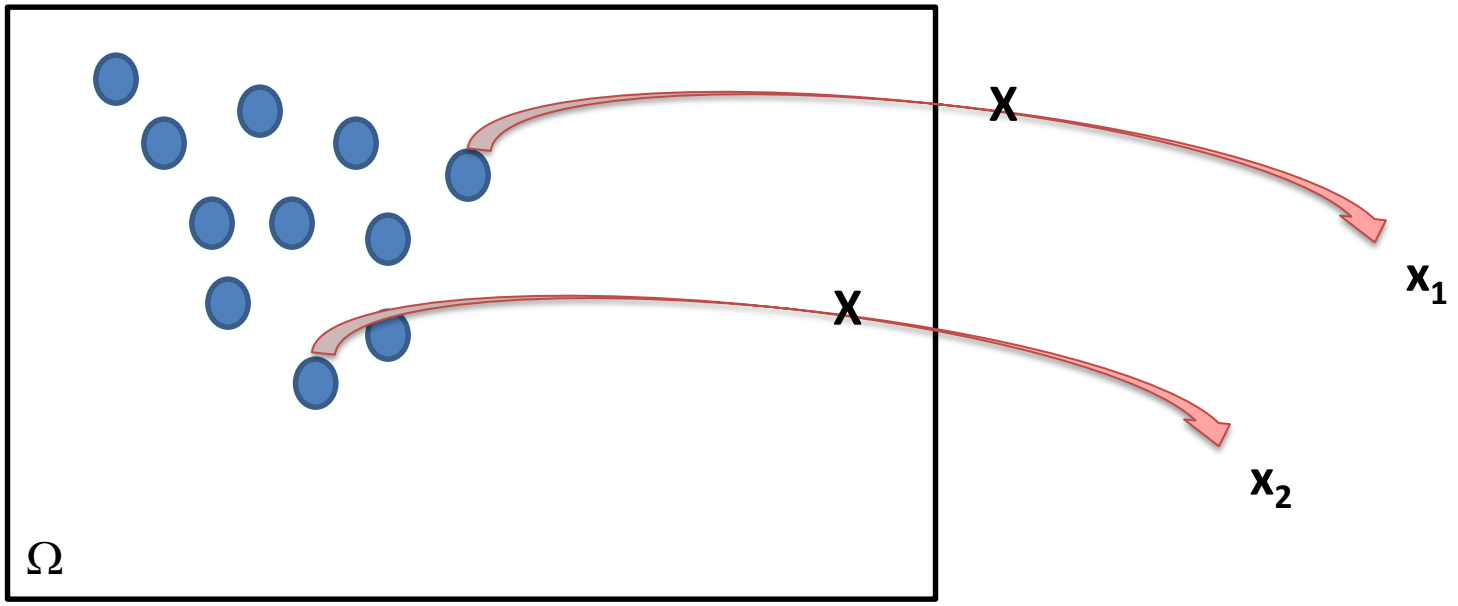
We can be clear that:

- Random variable is not random, is not a variable, an appropriate way to think about it:
- it's just a function from the sample space to the real numbers!

- They can be discrete or continuous
  - Round the height to the nearest centimeter (that's discrete)
  - Infinite precision weight recorder (that's continuous)

- We will start with discrete random variables

- Important concept fundamentally (seems trivial)
  - Random variable is a function (can be thought to be a subroutine in a programming language)
  - X: random variable
  - x: numerical values



We want to describe something about the relative likelihoods of the different numerical values that the random variable can take

We are asking: how likely is it that we obtain an outcome that leads to that particular numerical value

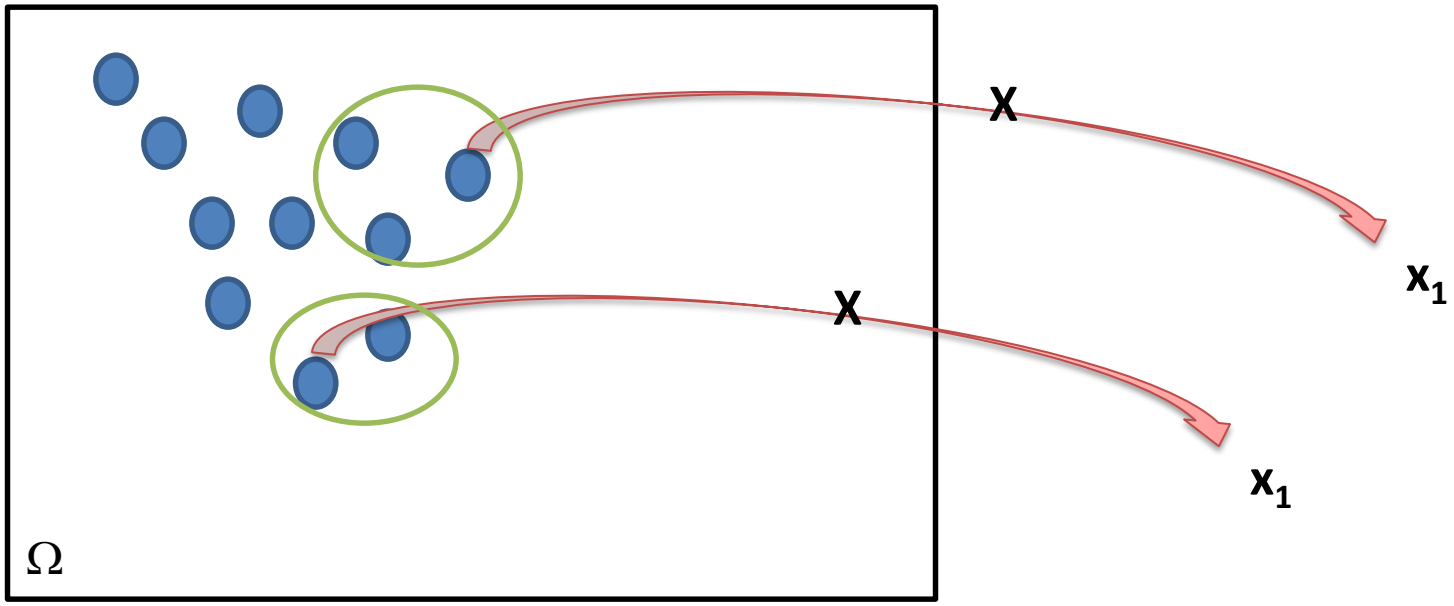
Probability mass function, a function describing how ‘mass’ is distributed on every value that random variable can take

Probability mass function (PMF)

- "probability law, probability distribution of X"
- Let's write out the notation of this probability

$$p_X(x) = P(X = x) = P(\{\omega \in \ \Omega, \text{s.t. } X(\omega) = x\}$$

- Since, these are probabilities:
  - $p_X(x) \geq 0$
  - $\sum_x p_X = 1$
- Example: X (random variable) is the number of coin tosses until first head
  - First toss, X =1, second toss X = 2, n^{th} toss X=n
  - Assume independent toss:
    - P(H) = p
    - $p_X(k) = P(X = k) = P(TT \ldots TH) = (1-p)^{k-1}p, \ \text{k} = 1,2, \ldots$


X
x_1
X
x_1
Ω