EE3030 Probability Team Problem (Group 5)

Q1:
If there is a special campaign is held in a mobile game, and the probability of getting rare cards would be raised. There is only SR card and SSR card in the campaign card pool, and the probability of obtaining them are 20% and 80% respectively. Let the draw time to be random variable X, and X is normal distributed with the mean to be , what is the probability of getting more than 20 SSR cards in 50 draws?

A1:
From the information given by the problem, we could know that:
E[x] = n*p = 50*0.2 = 10 = μ
σ^2 = n*p*(1-p) = 50*0.2*0.8 = 8
$\sigma = 8^{0.5}$

$$P(X > 20) = P\left(\frac{X-\mu}{\sigma} > \frac{20-10}{2\sqrt{2}}\right) = 1 - \Phi z(3.54)$$
$$= 1 - 0.9997922 = 0.0002078$$

Since random variable X is normal distributed, so we could calculate as:
As a result, the probability of getting more than 20 SSR cards is roughly equal to 0.02%

Q2:
A factory produces Xn gadgets on day n, where the Xn are independent and identically distributed random variables, with mean 5 and variance 9. Use Central Limit Theorem and the attached Gaussian table to solve the following problems.

(a) Find an approximation to the probability that the total number of gadgets produced in 100 days is less than 440.
(b) Find (approximately) the largest value of n such that
P(X1 + · · · + Xn ≥ 200 + 5n) ≤ 0.05.
(c) Let N be the first day on which the total number of gadgets produced exceeds 1000. Calculate an approximation to the probability that N ≥ 220.

A2:

(a) Let $W_N = \sum_{n=1}^{N} X_n$. Then
$P[\sum_{n=1}^{100} X_n \leq 440] = P[W_{100} \leq 440] \simeq \Phi(\frac{440-100*5}{\sqrt{9*100}}) = \Phi(-2) = 1 - \Phi(2) \simeq 0.02275$

(b) $P[W_n \geq 200 + 5n] = 1 - \Phi(\frac{200+5n-5n}{3\sqrt{n}})$
$1 - \Phi(\frac{200}{3\sqrt{n}}) \leq 0.05$
$\Phi(\frac{200}{3\sqrt{n}}) \geq 0.95$
$\frac{200}{3\sqrt{n}} \geq 1.65$
$n \leq 1632$

(c) $P[N \geq 220] = P[\sum_{n=1}^{219} X_n < 1000] = \Phi(\frac{1000-219*5}{3\sqrt{219}}) = \Phi(-2.1398) \simeq 0.016$

Q3:

Suppose the age of cars on the road is Normally distributed with a mean of 7.2 years. If the standard deviation is known to be 2.1 years, what is the probability that 12 randomly selected cars have been on the road for between 6 and 8 years?

A3:

$$\overline{x} \sim N\left(7.2, \frac{2.1}{\sqrt{12}}\right)$$

$$z_1 = \frac{6 - 7.2}{\frac{2.1}{\sqrt{12}}} = -1.98$$

$$z_2 = \frac{8 - 7.2}{\frac{2.1}{\sqrt{12}}} = 1.32$$

$$P(6 \leq \overline{x} \leq 8)$$

$$= P((-1.98 \leq z \leq 1.32)$$

$$= .9066 - .0239 = .8827$$

Q4:

Let the probability density function of a random variable X be

f(x) = (x^n *e^-x)/n! , x>=0

show that P(0 < X < 2n+2) > n/(n+1)

A4:

by integration
E[X] = n+1 , E[X^2] =(n+2)(n+1)
var(X) = n+1
then , use chebyshev inequality
P(|X-(n+1)| < n+1) > 1 - (var(x) / (n+1)^2)
so, P(0 < X < 2n+2) > n/(n+1)

Q5:

Prove that if $\text{Cov}(X, Y) = 0$, then

$$\rho(X + Y, X - Y) = \frac{\text{Var}(X) - \text{Var}(Y)}{\text{Var}(X) + \text{Var}(Y)}.$$

A5:

$$\text{Cov}(X + Y, X - Y) = \text{Var}(X) - \text{Var}(Y).$$

Since $\text{Cov}(X, Y) = 0$,

$$\begin{aligned}
\sigma_{X+Y} \cdot \sigma_{X-Y} &= \sqrt{\text{Var}(X + Y) \cdot \text{Var}(X - Y)} \\
&= \sqrt{\big[\text{Var}(X) + \text{Var}(Y)\big]\big[\text{Var}(X) + \text{Var}(Y)\big]} \\
&= \text{Var}(X) + \text{Var}(Y).
\end{aligned}$$

Therefore,

$$\rho(X + Y, X - Y) = \frac{\text{Cov}(X + Y, X - Y)}{\sigma_{X+Y} \cdot \sigma_{X-Y}} = \frac{\text{Var}(X) - \text{Var}(Y)}{\text{Var}(X) + \text{Var}(Y)}.$$