10720 EECS 303003 Probability Homework #4 Due: 5/10

1. Given the joint density function of X, Y as following

$$f_{XY}(x,y) = \begin{cases} \dfrac{2x+y}{a}, & 0 \le x \le 2, \quad 0 \le y \le 4 \\ 0 \quad , & \text{otherwise} \end{cases}$$

   (a) Find the value of a.
   (b) Find the covariance of X and Y, $Cov(x,y)$.
   (c) Find the correlation coefficient of X and Y, ρ_{XY}.

2. Suppose that n people have their hats returned at random. Let $X_i = 1$ if the ith person gets his/her own hat back and 0 otherwise. Let $S_n = \sum_{i=1}^{n} X_i$. Then S_n is the total number of people who get their own hats back. Please calculate the followings:
   (a) $E[X_i^2]$
   (b) $E[X_i \cdot X_j], i \neq j$
   (c) $E[S_n^2]$
   (d) $Var[S_n]$

3. Romeo and Juliet have a date at a given time, denote that random variable X and Y is the amount of time where Romeo and Juliet are late respectively. Assume X and Y are independent and exponentially distributed with different parameters λ and μ, respectively. Find the PDF of X – Y.

4. The random variables X, Y, and Z are independent and uniformly distributed between zero and one. Find the PDF of X + Y + Z .

5. Let X be a random variable that takes nonnegative integer values, and is associated with a transform of the form

$$M_x(s) = c\,\frac{3 + 4e^{2s} + 2e^{3s}}{3 - e^s}$$

   Where c is some scalar. Find E[X], $p_x(1)$, and $E[X|X \neq 0]$.