We define the joint density function of X and Y as :

$$f_{X,Y}(x,y)=\frac{1}{4}\ for\ (x,y)\in\{(0,0),(1,1),(1,-1),(2,0)\},\ 0\ otherwise$$

(a) Find the covariance of X and Y, $Cov(x,y)$.

(b) Find the correlation coefficient of X and Y, $\rho_{XY}$.

- $f_X(x)=\sum_{y} f_{X,Y}(x,y)=\begin{cases}1/4, & x=0,2\\ 1/2, & x=1\\ 0, & \text{otherwise}\end{cases}$

$E[X]=\sum_{x=0}^{2} x\, f_X(x)=\frac{1}{2}+\frac{2}{4}=1$

$E(X^2)=\sum_{x=0}^{2} x^2 f_X(x)=\frac{1}{2}+\frac{2^2}{4}=\frac{3}{2}$

$Var(X)=E(X^2)-E(X)^2=\frac{1}{2}$

- $f_Y(y)=\sum_{x} f_{X,Y}(x,y)=\begin{cases}1/4, & y=-1,1\\ 1/2, & y=0\\ 0, & \text{otherwise}\end{cases}$

$E(Y)=\sum_{y=-1}^{1} y\, f_Y(y)=-\frac{1}{4}+\frac{1}{4}+0=0$

$E(Y^2)=\sum_{y=-1}^{1} y^2 f_Y(y)=\frac{1}{4}+0+\frac{1}{4}=\frac{1}{2}$

$Var(Y)=E(Y^2)-E(Y)^2=\frac{1}{2}$

$\Rightarrow$ a) $Cov(x,y)=E[XY]-E(X)E(Y)$

and $E(XY)=\sum_{x=0}^{2}\sum_{y=-1}^{1} xy\, f_{XY}(x,y)=0\times\frac{1}{4}+1\times\frac{1}{4}+(1\times(-1))\times\frac{1}{4}+0\times\frac{1}{4}=0$

Hence $Cov(x,y)=0$

b) $\rho_{XY}=\frac{Cov(X,Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}}=0$