10720 EECS 303003 Probability Homework #3

- 1. Alvin throws darts at a circular target of radius r and is equally likely to hit any point in the target. Let X be the distance of Alvin's hit from the center.
 - (a) Find the PDF, the mean. and the variance of X.
 - (b) The target has an inner circle of radius t. If $X \le t$, Alvin gets a score of S = 1/X. Otherwise his score is S = 0. Find the CDF of S. If S a continuous random variable?
- 2. Let the random variable X be uniform distribution in the union of intervals (0, a) and (a+2, b). Let F be the CDF of X. Assume that F(4) = 0.2 and F(a + 1) = 0.25.
 - (a) Find a and b
 - (b) Find F(8.39)
 - (c) Find $F(3.01 \le X \le 9.14)$
- 3. The joint probability density function of random variables X and Y is given by

$$f_{XY}(x, y) = \begin{cases} \frac{k}{\pi} e^{-\frac{x^2 + y^2}{2}}, & xy \ge 0\\ 0, & xy < 0 \end{cases}$$

- (a) Find k
- (b) Find marginal PDFs $f_X(x)$ and $f_Y(y)$
- (c) Are X and Y independent? Please explain it.
- 4. Let X and Y be continuous random variables having joint density function given by

$$f_{XY}(x,y) = \begin{cases} n(n-1)(y-x)^2, & 0 \le x \le y \le 1\\ 0, & \text{elsewhere} \end{cases}$$

(a) Find conditional expectation of Y given X = x, i.e., E[Y|X]

- (b) Please use answer of (a) to compute expectation of Y, i.e., E[Y]
- (c) Show and Prove the relationship between E[Y] and E[Y|X]
- 5. Let X and Y be independent random variables with PDF $f_X(x) = \lambda e^{-\lambda x}$; $x \ge 0$ and $f_Y(y) = \lambda e^{-\lambda y}$; $y \ge 0$, Find PDF of Z = X + Y

6. (Bonus) Let X be the input of communication channel and takes the values 1 with probability p and -1 with probability (1 - p). Suppose that the output of the channel is Y = X + M, where M is a Laplacian random variable with probability density function $f_M(m) = \frac{1}{2}\alpha e^{-\alpha|m|}; -\infty < m < \infty$. Find the probability density function of Y.