EE3060 Probability – Proposed questions and answers

GROUP 5

HW#3

Question 1:

Let the continuous random variables (X,Y) have joint density function f(x,y) = e^-y ,0<x<y<∞, calculate P(X+Y>=1)

Answer 1:

P(X+Y>=1) = 1-P(X+Y<1) = 1-(1- 2e^-0.5 +e^-1) = 2e^-0.5 - e^-1

Question 2:

The joint probability density function of X and Y is given by

$$f(x,y)=\frac{6}{7}(x^2+\frac{xy}{2}),\ \ 0<x<1, 0<y<2$$

What is the density function of X?

Answer 2:

$$f_X(x) = \int_0^2 f(x,y)dy$$

$$= \int_0^2 6/7(x^2+xy/2)dy$$

$$= 6/7\,[x^2 y + xy^2/4]_0^2$$

$$\mathbf{f_X(x) = 6/7\,[2x^2 + x]}$$

Question 3:

In a shooting contest, there is a target is placed at a proper distance from the competitors. The target is a square with an inner circle, let the side length of square is k, the radius of the inner circle is r. ($r \le 0.5k$) Still, the center of the inner circle and square are same. Let X be the distance from the center, please find the CDF and PDF. (hint: you might find the CDF first, then find the PDF based on CDF)

Answer 3:

First, we calculate the CDF by the area of target:

$X \le r$:

$F(x) = P(X \le r) = \pi r^2/\pi x^2 = r^2/x^2$

$r < X \le 0.5k$:

$F(X) = P$ (hit at the region of square, but not in the circle)

$= (k^2 - \pi r^2)/\pi x^2$

$X > 0.5k$:

$F(X) = 0$

As a result, we could find the CDF.

Second, we want to find the PDF of it, we have to remember that if we differentiate the CDF, then it will be changed into PDF, so we could know that:

$X \le r$:

$f(x) = d(F(X)) / dx = -2r^2/x$

$r < X \le 0.5k$:

$f(x) = d(F(X)) / dx = -2*[(k^2 - \pi r^2) / \pi x]$

$X > 0.5k$:

$f(x) = d(F(X)) / dx = 0$

Consequently, we find the PDF of it.

Question 4:

Let X be uniformly distributed in the unit interval [0, 1] . Consider the random variable Y = g(X), where

$$g(x) = \begin{cases} 1, & \text{if } x \le 1/3, \\ 2, & \text{if } x > 1/3. \end{cases}$$

Find the expected value of Y by first deriving its PMF. Verify the result using the expected value rule++++.

Answer 4:

The random variable Y = g(X) is discrete and its PMF is given by

$$p_Y(1) = \mathbf{P}(X \leq 1/3) = 1/3, \qquad p_Y(2) = 1 - p_Y(1) = 2/3.$$

Thus,

$$\mathbf{E}[Y] = \frac{1}{3} \cdot 1 + \frac{2}{3} \cdot 2 = \frac{5}{3}.$$

The same result is obtained using the expected value rule:

$$\mathbf{E}[Y] = \int_0^1 g(x) f_X(x)\, dx = \int_0^{1/3} dx + \int_{1/3}^1 2\, dx = \frac{5}{3}.$$

Question 5:

Zwei is a naughty dog with his kennel located at the origin of a Cartesian coordinate system. His breeder, Ruby, finds out that the probability for finding Zwei at position (x,y) follows a joint probability density function,

$$f_{X,Y}(x, y) = \frac{1}{2\pi} e^{-\frac{x^2+y^2}{2}}$$

We define the territory of a dog as a circular area where we could find that dog with probability 90%, and the center of the area is located at the kennel of that dog.

What is the radius R for the territory of Zwei?

(assume ln(10)=2.3)

Answer 5:

check: $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{2\pi} e^{-\frac{x^2+y^2}{2}} dydx = 1$ ok

<i> CDF: $F(R) = \int_0^R \int_0^{2\pi} \frac{1}{2\pi} e^{-\frac{r^2}{2}} r d\theta dr$ $\quad r^2 = x^2+y^2$

$$= \frac{1}{2\pi}\int_0^{2\pi} d\theta \cdot \int_0^R e^{-\frac{r^2}{2}} \cdot r dr$$

$$= -e^{-\frac{r^2}{2}} \Big|_0^{r=R}$$

$$= 1 - e^{-\frac{R^2}{2}}$$

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$$F(R) = 1 - e^{-\frac{R^2}{2}} = 0.9$$

$$\Rightarrow e^{-\frac{R^2}{2}} = 0.1$$

$$\Rightarrow -\frac{R^2}{2} = -\ln 10 = -2.3$$

$$\Rightarrow R = \sqrt{4.6}$$

Ans: $\sqrt{4.6}$