Probability Homework 2 DUE: 3/27 21:00

### Problem 1

Fischer and Spassky play a chess match in which the first player to win a game wins the match. After 10 successive draws, the match is declared drawn. Each game is won by Fischer with probability 0.4 is won by Spassky with probability 0.3.

- (a) What is the probability that Fischer wins the match?
- (b) What is the PMF of the duration of the match?

#### Problem 2

A packet communication system consists of a buffer that stores packets from some source, and a communication line that retrieves packets from the buffer and transmits them to a receiver. The system operates in time-slot pairs. In the first slot, the system stores a number of packets that are generated by the source according to a Poisson PMF with parameter  $\lambda$ ; however, the maximum number of packets that can be stored is a given integer b, and packets arriving to a full buffer are discarded. In the second slot, the system transmits either all the stored packets or c packets (whichever is less). Here, c is a given integer with 0 < c < b.

(a) Assuming that at the beginning of the first slot the buffer is empty, find the PMF of the number of packets stored at the end of the first slot and at the end of the second slot.(b) What is the probability that some packets get discarded during the first slot?

#### **Problem 3**

Let X be a random variable with PMF

$$p_X(x) = \begin{cases} \frac{x^2}{a}, & \text{if } x = -3, -2, -1, 0, 1, 2, 3\\ 0, & \text{otherwise} \end{cases}$$

(a) Find a and E[X].

(b) What is the PMF of the random variable  $Z = (X - E[X])^2$ ?

- (c) Using the result from (b), find the variance of X.
- (d) Find the variance of X using the formula  $var(X) = \sum_{x} (x E[X])^2 p_X(x)$ .

# **Problem 4**

(a) A fair coin is tossed repeatedly and independently until two consecutive heads or two consecutive tails appear. Find the PMF, the expected value, and the variance of the number of tosses.

(b) Assume now that the coin is tossed until we obtain a tail that is immediately preceded by a head. Find the PMF and the expected value of the number of tosses.

## Problem 5

Consider 2m persons forming m couples who live together at a given time. Suppose that at some later time, the probability of each person being alive is p, independent of other persons. At that later time. let A be the number of persons that are alive and let S be the number of couples in which both partners are alive. For any survivor number a, find E[S|A = a].